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Convex Analysis and Minimization Algorithms I

Fundamentals

With 113 Figures

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Introduction

During the French Revolution, the writer of a project of law on public instruction complained: “Le défaut ou la disette de bons ouvrages élémentaires a été, jusqu'à présent, un des plus grands obstacles qui s'opposaient au perfectionnement de l'instruction. La raison de cette disette, c'est que jusqu'à présent les savants d'un mérite éminent ont, presque toujours, *préféré la gloire d'élever l'édifice de la science à la peine d'en éclairer l'entrée.*[1]” Our main motivation here is precisely to “light the entrance” of the monument Convex Analysis and Minimization Algorithms. This is therefore not a reference book, to be kept on the shelf by an expert who already knows the building and can find his way through it; it is rather a book for the purpose of learning and teaching. We call above all on the intuition of the reader, and our approach is very gradual: several developments are made first in a simplified context, and then repeated in subsequent chapters at a more sophisticated level. Nevertheless, we keep constantly in mind the minimization problem suggested by A. Einstein: “Everything should be made as simple as possible, but not simpler”. Indeed, the content is by no means elementary, and will be hard for a reader not possessing a firm mastery of basic mathematical skill.

As suggested by the title, two distinct parts are involved. One, convex analysis, can be considered as an academic discipline, of a high pedagogical content, and is potentially useful to many. Minimization algorithms, on the other hand, form a much narrower subject, definitely concerning applications of mathematics, and to some extent the exclusive domain of a few specialists. Besides, we restrict ourselves to what is called nonsmooth optimization, and even more specifically to the so-called bundle algorithms. These form an important application of convex analysis, and here lies an incentive to write the present bi-disciplinary book. The theory is thus illustrated with a typical field of applications, and in return, the necessary mathematical background is thus accessible to a reader more interested by the algorithmic part. This has some consequences for the expository style: for the theoretical part, the pedagogy is based on geometric visualization of the mathematical concepts; as for minimization, only a vague knowledge of computers and numerical algorithms is assumed of the reader, which implies a rather pedestrian pace here and there.

[1]“The lack or scarcity of good, elementary books has been, until now, one of the greatest obstacles in the way of better instruction. The reason for this scarcity is that, until now, scholars of great merit have almost always preferred the glory of constructing the monument of science over the effort of lighting its entrance.” D. Guedj: *La Révolution des Savants*, Découvertes, Gallimard Sciences (1988) 130 – 131.

This dichotomous aspect emerges already in the first two chapters, which make a quick guided tour of their respective fields. Many a reader might be content with Chap. I, in which most concepts are exposed (extended-valued functions, subdifferentiability, conjugacy) in the simplest setting of univariate functions. As for Chap. II, it can be skipped by a reader familiar with classical minimization algorithms: its aim is to outline the general principles which, in our opinion, nonsmooth optimization must start from, and such a reader knows these principles.

Chapters III to VI are the instructional backbone of the work. Entirely devoted to convex analysis, they contain the basic theory, and geometric intuition is involved more than anywhere else. Chapter VII does the same thing for basic optimization theory.

Finally the last chapter of the present first part (Chap. VIII) lays down the necessary theory to develop algorithms minimizing convex functions. This chapter follows the general principles of Chap. II and serves as an illustration of basic convex analysis. On the other hand, its material is essential for a comprehension of the actual algorithms for convex (nonsmooth) optimization, to be studied in the second part.

Each chapter is presented as a "lesson", in the sense of our old masters, treating of a given subject in its entirety. We could not completely avoid references to other chapters; but for many of them, the motivation is to suggest an intellectual link between apparently independent concepts, rather than a technical need for previous results. More than a tree, our approach evokes a spiral, made up of loosely interrelated elements.

Formally, many sections are written in smaller characters; these are not reserved to advanced material. Actually, these sections often help the reader, with illustrative examples, side remarks helping to understand a delicate point, or preparing some material to come in a subsequent chapter. Roughly speaking, they can be compared to footnotes, used to avoid interrupting the flow of the development; it can be helpful to skip them during a deeper reading, with pencil and paper. There are no formally stated exercises; but these sections in smaller characters, precisely, can often be considered as such exercises, useful to keep the reader awake.

The numbering restarts at 1 in each chapter, and chapter numbers are dropped in a cross-reference to an equation or theorem from within the same chapter. A reference of the type A.n refers to Appendix A, which recalls some theoretical background.

We thank all those, including the referees, who contributed the improvement of the manuscript by their remarks, criticisms or suggestions. Mistakes? there still must be some, of course: we just hope that they are no longer capital, and that readers will be able to detect and correct them painlessly.

Among those who helped us most, we would like to thank particularly Th. Dussaut, J.C. Gilbert, K.C. Kiwiel, S. Maurin, J.-J. Moreau, A.S. Nemirovskij, M.-R. Philippe, C.A. Sagastizábal, A. Seeger, S. Shiraishi, M. Valadier and, last but not least, the editorial and production staff of Springer-Verlag, who did a remarkably professional job. The manuscript was written on an Apple Mac+, using Microsoft Word, and CricketDraw for the pictures. It was converted into TeX with the help of "rtf2TeX", a program written by R. Lupton at Princeton University. The final typeset version was

produced using the MathTime fonts by M. Spivak, distributed by the TeXplorators Corp. The role of OzTeX was decisive in this, and we gratefully acknowledge the technical help of W. Carlip and A. Trevorrow. Thanks and apologies are also due to Thérèse, Lydie, Sébastien, Aurélien, who had to endure our bad mood during seven years of wrestling with mathematics, computers and the English language.

Toulouse, April 1993 J.-B. **H**iriart-**U**rruty, C. Lemaréchal

Note about this revised printing. Most corrections are minor; they concern misprints and other typographical details, or also informal developments. Besides, some bibliographical items have been updated and the index has been enriched.

Paris, January 1996