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PREFACE

This volume contains 18 papers by members and guests of the former Sonderforschungsbereich 72 (SFB 72) "Approximation und Mathematische Optimierung in einer anwendungsbezogenen Mathematik", who have, over the years, collaborated in the research group "Lösung partieller Differentialgleichungen und Variationsrechnung". It is an account of 15 years of research.

The SFB 72 was in existence from 1971 to 1986 as an institute at the University of Bonn. It was supported by the Deutsche Forschungsgemeinschaft, by the University, and by the State of Nordrhein-Westfalen. Members of the special research group were mathematicians holding chairs at Bonn University as well as younger scientists with temporary positions. They collaborated with colleagues from neighbouring universities and with numerous guests from abroad who visited the SFB 72.

In 1986 the SFB 72 reached its maximum life span of 15 years. On this occasion we invited our colleagues and our former guests to contribute a research paper or a survey of a part of our common field of interest, the theory of partial differential equations and the calculus of variations. We hope to have attained a good balance of surveys and of original contributions. The emphasis is on existence and regularity results for variational problems and for nonlinear differential equations, on special equations of mathematical physics, and on problems of scattering theory.

We would like to take this opportunity to thank all those who have over the years supported the work of the Sonderforschungsbereich 72. Our special gratitude goes to the Deutsche Forschungsgemeinschaft and its referees. We are also deeply indebted to the State of Nordrhein-Westfalen, to our University, to our former colleagues and coworkers, and to our secretaries. We particularly thank Mrs. Anke Vogt who, as always, has overcome all difficulties in preparing this volume.

Last but not least we thank the series editors and the publisher for including our celebration issue in the series *Lecture Notes in Mathematics* of Springer-Verlag.

Bonn, 15th April 1988

Stefan Hildebrandt
Rolf Leis

TABLE OF CONTENTS

| | Page |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| Alber, Hans Dieter: On the existence in the large of solutions to the one-dimensional, isentropic hydrodynamic equations in a bounded domain | 1 |
| Alber, Hans Dieter & Leis, Rolf: Initial boundary value and scattering problems in mathematical physics | 23 |
| Alt, Hans Wilhelm: On shape optimization of a turbine blade | 61 |
| Bemelmans, Josef: Free boundary problems for the Navier-Stokes equations | 96 |
| Dierkes, Ulrich: A geometric maximum principle, Plateau's problem for surfaces of prescribed mean curvature, and the two dimen- sional analogue of the catenary | 116 |
| Dziuk, Gerhard: Finite elements for the Beltrami operator on arbitrary surfaces | 142 |
| Finn, Robert: Comparison principles in capillarity | 156 |
| Frehse, Jens: Remarks on diagonal elliptic systems | 198 |
| Giaquinta, Mariano: Quasiconvexity, growth conditions and partial regularity | 211 |
| Grüter, Michael: The monotonicity formula in geometric measure theory, and an application to a partially free boundary problem | 238 |
| Gulliver, Robert: Isoperimetric problems having continua of solutions | 256 |
| Jost, Jürgen: Harmonic maps - analytic theory and geometric significance | 264 |
| Meier, Michael: Asymptotic behavior of solutions of some quasilinear elliptic systems in exterior domains | 297 |
| Milani, Albert & Picard, Rainer: Decomposition theorems and their application to non-linear electro- and magneto-static boundary value problems | 317 |

| | Page |
|----------------------------------------------------------------------------------------------------------------------------|------|
| Racke, Reinhard: Initial boundary value problems in thermoelasticity | 341 |
| Struwe, Michael: Applications of variational methods to problems in the geometry of surfaces | 359 |
| Tromba, Anthony: Open problems in the degree theory for disc minimal surfaces spanning a curve in \mathbb{R}^3 | 379 |
| Witsch, Karl-Josef: On a modified version of the free geodetic boundary-value problem | 402 |