

Contents

Preface *vii*

1 Sets and Functions **1**

- 1. Sets* 2
- 2. Sets and Counting* 8
- 3. Functions* 19

2 Number Theory **33**

- 1. Prime Numbers* 34
- 2. Some Formal Aspects* 45
- 3. Some Formal Consequences* 49
- 4. Some Basic Properties* 56
- 5. Equivalence Relations* 65
- 6. Congruence* 69
- 7. Applications of the Pigeon-Hole Principle* 79
- 8. Waring's Problem* 84
- 9. Fermat and Mersenne Primes* 89

3 Permutations 99

1. *The Set $A(S)$* 101
2. *Cycle Decomposition* 102
3. *Even and Odd* 110
4. *The Interlacing Shuffle* 118
5. *The Josephus Permutation* 121
- Bibliography* 127

4 Group Theory 129

1. *Definition and Examples of Groups* 130
2. *Some Beginning Notions and Results* 133
3. *Subgroups* 136
4. *Lagrange's Theorem* 141
5. *Isomorphism* 146

5 Finite Geometry 153

1. *Introduction* 153
2. *Affine Planes* 156
3. *Counting Arguments* 160
4. *Planes of Low Order* 166
5. *Coordinate Affine Planes* 170
6. *Parallelograms and Midpoints* 176
7. *The Nonexistence of Planes of Order 6* 184
- Suggestions for Further Reading* 193

6 Game Theory 195

1. *Probability* 196
2. *Mathematical Expectation* 198
3. *Preliminary Remarks on Games* 199
4. *Examples* 201
5. *The General Two-by-Two Game* 204
6. *Simplified Poker* 208
7. *The Game Without a Name* 212
8. *Goofspiel* 215
- Suggestions for Further Reading* 219

7 Infinite Sets 221

1. *Infinite Numbers; Countable Sets* 221
2. *Uncountable Sets* 234