

**Jiří Herman Radan Kučera
Jaromír Šimša**

Equations and Inequalities

**Elementary Problems and
Theorems in Algebra and
Number Theory**

Translated by Karl Dilcher



Springer

Jiří Herman
Brno Grammar School
tř. Kpt. Jaroše 14
658 70 Brno
Czech Republic

Jaromír Šimša
Mathematical Institute
Academy of Sciences of the Czech
Republic
Žižkova 22
616 62 Brno
Czech Republic

Editors-in-Chief
Rédacteurs-en-chef

Jonathan Borwein
Peter Borwein
Centre for Experimental and Constructive Mathematics
Department of Mathematics and Statistics
Simon Fraser University
Burnaby, British Columbia V5A 1S6
Canada

Radan Kučera
Department of Mathematics
Faculty of Science, Masaryk University
Janáčkovo nám. 2a
662 95 Brno
Czech Republic

Translator
Karl Dilcher
Department of Mathematics and Statistics
Dalhousie University
Halifax, Nova Scotia B3H 3J5
Canada

Mathematics Subject Classification (2000): 16G20, 20Kxx

Library of Congress Cataloging-in-Publication Data
Herman, Jiří.

[Metody řešení matematických úloh I. English]

Equations and inequalities : elementary problems and theorems in algebra and number theory / Jiří Herman, Radan Kučera, Jaromír Šimša ; translated by Karl Dilcher.

p. cm. — (CMS books in mathematics)

Includes bibliographical references and index.

ISBN 978-1-4612-7071-3 ISBN 978-1-4612-1270-6 (eBook)

DOI 10.1007/978-1-4612-1270-6

1. Equations—Numerical solutions. 2. Inequalities. 3. Problem solving. I. Kučera Radan. II. Šimša, Jaromír. III. Title. IV. Series.

QA218.H4713 2000

512.9'4—dc21

99-047384

Printed on acid-free paper.

© 2000 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 2000
Softcover reprint of the hardcover 1st edition 2000

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Allan Abrams; manufacturing supervised by Jerome Basma.
Photocomposed copy prepared from the translator's \LaTeX files.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-7071-3

Contents

Preface	v
Symbols	xi
1 Algebraic Identities and Equations	1
1 Formulas for Powers	1
2 Finite Sums	5
3 Polynomials	23
4 Symmetric Polynomials	38
5 Systems of Equations	46
6 Irrational Equations	60
7 Some Applications of Complex Numbers	75
2 Algebraic Inequalities	89
1 Definitions and Properties	90
2 Basic Methods	94
3 The Use of Algebraic Formulas	106
4 The Method of Squares	113
5 The Discriminant and Cauchy's Inequality	124
6 The Induction Principle	135
7 Chebyshev's Inequality	145
8 Inequalities Between Means	150
9 Appendix on Irrational Numbers	169

3	Number Theory	173
1	Basic Concepts	174
2	Prime Numbers	183
3	Congruences	189
4	Congruences in One Variable	201
5	Diophantine Equations	217
6	Solvability of Diophantine Equations	238
7	Integer Part and Fractional Part	251
8	Base Representations	258
9	Dirichlet's Principle	268
10	Polynomials	274
4	Hints and Answers	283
1	Hints and Answers to Chapter 1	283
2	Hints and Answers to Chapter 2	294
3	Hints and Answers to Chapter 3	312
	Bibliography	337
	Index	341