

**Jiří Herman      Radan Kučera**  
**Jaromír Šimša**

# **Counting and Configurations**

**Problems in Combinatorics,  
Arithmetic, and Geometry**

Translated by Karl Dilcher

With 111 Figures



**Springer**

Jiří Herman  
Brno Grammar School  
tř. Kpt. Jaroše 14  
658 70 Brno  
Czech Republic

Jaromír Šimša  
Mathematical Institute  
Academy of Sciences of the  
Czech Republic  
Žižkova 22  
616 62 Brno  
Czech Republic

*Editors-in-Chief*

*Rédacteurs-en-chef*

Jonathan Borwein

Peter Borwein

Centre for Experimental and Constructive Mathematics

Department of Mathematics and Statistics

Simon Fraser University

Burnaby, British Columbia V5A 1S6

Canada

cbs-editors@cms.math.ca

Radan Kučera  
Department of Mathematics  
Faculty of Science  
Masaryk University  
Janáčkovo nám. 2a  
662 95 Brno  
Czech Republic

*Translator*

Karl Dilcher

Department of Mathematics  
and Statistics

Dalhousie University

Halifax, Nova Scotia B3H 3J5

Canada

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