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An Outline of Set Theory

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James M. Henle
Department of Mathematics
Smith College
Northampton, MA 01060
U.S.A.

Series Editor

Paul R. Halmos
Department of Mathematics
University of Santa Clara
Santa Clara, CA 95053
U.S.A.

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Preface

This book is designed for use in a one semester problem-oriented course in undergraduate set theory. The combination of level and format is somewhat unusual and deserves an explanation.

Normally, problem courses are offered to graduate students or selected undergraduates. I have found, however, that the experience is equally valuable to ordinary mathematics majors. I use a recent modification of R. L. Moore's famous method developed in recent years by D. W. Cohen [1]. Briefly, in this new approach, projects are assigned to groups of students each week. With all the necessary assistance from the instructor, the groups complete their projects, carefully write a short paper for their classmates, and then, in the single weekly class meeting, lecture on their results. While the emphasis is on the student, the instructor is available at every stage to assure success in the research, to explain and critique mathematical prose, and to coach the groups in clear mathematical presentation.

The subject matter of set theory is peculiarly appropriate to this style of course. For much of the book the objects of study are familiar and while the theorems are significant and often deep, it is the methods and ideas that are most important. The necessity of reasoning about numbers and sets forces students to come to grips with the nature of proof, logic, and mathematics. In their research they experience the same dilemmas and uncertainties that faced the pioneers. They will, for example, discover in some chapters that deeper results in earlier chapters are necessary before work can proceed.

Students do not always solve the problems completely on their own. They do, however, learn what proofs are and how to organize and write them, and while lectures on this material might easily bore, students find the experience of doing it themselves exciting and rewarding. It is familiar enough to be reassuring and different enough to be challenging.

More set theory is included here than one can reasonably use. I cover roughly 35 to 40 projects in a semester. The last three chapters are independent of each other and can be used selectively or omitted. Sections of other chapters may also be skipped or summarized, particularly the last few in Chapter 7.

I am indebted first of all to David Cohen, for the example of his outstanding teaching, and to my students for their intelligence and unflagging good humor. I only hope that my confidence in this approach is not based entirely on a teacher who might succeed with *any* method, and students who might prevail under *any* regimen. I greatly appreciate the support of Smith College and the encouragement of its most collegial mathematics department. Thanks are also due to Marcia Groszek for the Tennyson quotation, and special thanks to Carlos Di Prisco for his very timely suggestions and advice.

References

[1] D. W. Cohen, "A Modified Moore Method for Teaching Undergraduate Mathematics," *Am. Math. Monthly* **89**, no. 7, 1982.

J. M. HENLE

Contents