

Mathematical Visualization

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Konrad Polthier (Eds.)

Mathematical Visualization

Algorithms, Applications
and Numerics

With 187 Figures, 46 in Color
and 12 Tables



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Preface

Mathematical Visualization is a young field in the interdisciplinary area of numerics, geometry, and computer graphics. It develops powerful visualization tools for mathematical research and utilizes mathematical techniques for computer graphics and scientific visualization.

The present book is the second in a series of publications on this subject. The articles were presented at the international workshop "Visualization and Mathematics", held from September 16-19, 1997 in Berlin-Dahlem (Germany). Well-known experts contributed latest research material to this volume. Each paper was carefully reviewed and evaluated by an international program committee. The articles cover many topics of mathematical visualization, comprising computer graphical techniques and visualization methods, handling of meshes and polygonal data representations, as well as application of visualization techniques in geometry and numerics.

We organized the material in the following five sections although many articles can not uniquely be associated with a single category:

- Meshes, Multilevel Approximation, and Visualization
- Geometry and Numerics
- Graphics Algorithms and Implementations
- Geometric Visualization Techniques
- Vector Fields and Flow Visualization.

The themes represent most active research topics. Specifically there are new methods and experimental results for surfaces with given curvature properties, the use of Morse theory in the validation of triangle nets, and Clifford algebras for approximation of vector fields. Promising trends are new developments in the numerics on discrete geometries, the study of adaptive and hierarchical techniques in space and time, and new visualization methods for displaying mathematical structures.

We hope the book unveils new insight into the evolving and fascinating area, and the reader will become acquainted with recent developments.

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