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Emmanuel Hebey

Sobolev Spaces on Riemannian Manifolds

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Author

Emmanuel Hebey
Département de Mathématiques
Université de Cergy-Pontoise
2 Avenue Adolphe Chauvin
F-95302 Cergy-Pontoise Cedex, France
e-mail: hebey@u-cergy.fr

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Introduction

This monograph is devoted to the study of Sobolev spaces in the general setting of Riemannian manifolds. In addition to being very interesting mathematical structures in their own right, Sobolev spaces play a central role in many branches of mathematics. While analysis proves more and more to be a very powerful means for solving geometrical problems, it is striking that no global study of these spaces exists in the general context of Riemannian manifolds. The objective of this monograph is to fill this gap, at least partially. In so doing, it is intended to serve as a textbook and reference for graduate students and researchers. This monograph also hopes to convince the reader that the naive idea that what is valid for Euclidean spaces must be valid for Riemannian manifolds is completely false. Indeed, as one will see, several surprising phenomena appear when studying Sobolev spaces in the Riemannian context. Elementary questions now give rise to sophisticated developments, where the geometry of the manifolds plays a central role. This monograph is full of such examples.

In a certain sense, Sobolev spaces are studied here for their own interest. Needless to say, they are fundamental in the study of PDE's. A striking example where they have played a major role in the Riemannian context is given by the famous Yamabe problem. The concept of best constants appeared there as crucial for solving limiting cases of some partial differential equations. (Geometric problems often lead to limiting cases of known problems in analysis). While the theory of Sobolev spaces for (non compact) manifolds has its origin in the 70's with the works of Aubin and Cantor, many of the results presented in this monograph have been obtained in the '80's and '90's. As the reader will easily be convinced, the study of Sobolev spaces in the Riemannian context is a field currently undergoing great development !

This monograph presupposes a preliminary course in Riemannian geometry. Not much is assumed to be known so that chapter 1 of Aubin [Au6] should provide specialists in analysis who do not know Riemannian geometry with sufficient knowledge for what follows. Needless to say, many excellent books on Riemannian geometry exist. Although the following ones are not the only possible quality choices, we refer the reader to Chavel [Ch], Gallot-Hulin-Lafontaine [GaHL], Jost [Jo], Kobayashi-Nomizu [KoN], and Spivak [Sp] for more details on what is assumed to be known here.

The material is organized into five chapters and several new results are presented. More precisely, the plan of this monograph is as follows.

Chapter 1 is devoted to the presentation of recent developments of Anderson and Anderson-Cheeger concerning harmonic coordinates, as well as the presentation of a packing result that will be often used in the following chapters.

Chapter 2 is devoted to the presentation of Sobolev spaces on Riemannian manifolds, and to the study of density problems.

Chapter 3 is devoted to Sobolev embeddings. This includes the presentation of general results on the topic, and the study of Sobolev embeddings for Euclidean spaces, compact manifolds, and complete manifolds.

Chapter 4 is devoted to what is currently called the best constants problems. Several results are discussed here, including those concerning the resolution of Aubin's conjecture by Hebey-Vaugon.

Finally, chapter 5 is devoted to the study of the influence of symmetries on Sobolev embeddings.

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Emmanuel Hebey
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