

## Contents

	<i>Preface</i>	<i>xi</i>
<b>1</b>	<b>Guiding problems</b>	<b>1</b>
	1.1 Implicitization	1
	1.2 Ideal membership	4
	1.3 Interpolation	5
	1.4 Exercises	8
<b>2</b>	<b>Division algorithm and Gröbner bases</b>	<b>11</b>
	2.1 Monomial orders	11
	2.2 Gröbner bases and the division algorithm	13
	2.3 Normal forms	16
	2.4 Existence and chain conditions	19
	2.5 Buchberger's Criterion	22
	2.6 Syzygies	26
	2.7 Exercises	29
<b>3</b>	<b>Affine varieties</b>	<b>33</b>
	3.1 Ideals and varieties	33
	3.2 Closed sets and the Zariski topology	38
	3.3 Coordinate rings and morphisms	39
	3.4 Rational maps	43
	3.5 Resolving rational maps	46
	3.6 Rational and unirational varieties	50
	3.7 Exercises	53
<b>4</b>	<b>Elimination</b>	<b>57</b>
	4.1 Projections and graphs	57
	4.2 Images of rational maps	61
	4.3 Secant varieties, joins, and scrolls	65
	4.4 Exercises	68

<b>5</b>	<b>Resultants</b>	73
5.1	Common roots of univariate polynomials	73
5.2	The resultant as a function of the roots	80
5.3	Resultants and elimination theory	82
5.4	Remarks on higher-dimensional resultants	84
5.5	Exercises	87
<b>6</b>	<b>Irreducible varieties</b>	89
6.1	Existence of the decomposition	90
6.2	Irreducibility and domains	91
6.3	Dominant morphisms	92
6.4	Algorithms for intersections of ideals	94
6.5	Domains and field extensions	96
6.6	Exercises	98
<b>7</b>	<b>Nullstellensatz</b>	101
7.1	Statement of the Nullstellensatz	102
7.2	Classification of maximal ideals	103
7.3	Transcendence bases	104
7.4	Integral elements	106
7.5	Proof of Nullstellensatz I	108
7.6	Applications	109
7.7	Dimension	111
7.8	Exercises	112
<b>8</b>	<b>Primary decomposition</b>	116
8.1	Irreducible ideals	116
8.2	Quotient ideals	118
8.3	Primary ideals	119
8.4	Uniqueness of primary decomposition	122
8.5	An application to rational maps	128
8.6	Exercises	131
<b>9</b>	<b>Projective geometry</b>	134
9.1	Introduction to projective space	134
9.2	Homogenization and dehomogenization	137
9.3	Projective varieties	140
9.4	Equations for projective varieties	141
9.5	Projective Nullstellensatz	144
9.6	Morphisms of projective varieties	145
9.7	Products	154
9.8	Abstract varieties	156
9.9	Exercises	162

CONTENTS		ix
<b>10</b>	<b>Projective elimination theory</b>	169
	10.1 Homogeneous equations revisited	170
	10.2 Projective elimination ideals	171
	10.3 Computing the projective elimination ideal	174
	10.4 Images of projective varieties are closed	175
	10.5 Further elimination results	176
	10.6 Exercises	177
<b>11</b>	<b>Parametrizing linear subspaces</b>	181
	11.1 Dual projective spaces	181
	11.2 Tangent spaces and dual varieties	182
	11.3 Grassmannians: Abstract approach	187
	11.4 Exterior algebra	191
	11.5 Grassmannians as projective varieties	197
	11.6 Equations for the Grassmannian	199
	11.7 Exercises	202
<b>12</b>	<b>Hilbert polynomials and the Bezout Theorem</b>	207
	12.1 Hilbert functions defined	207
	12.2 Hilbert polynomials and algorithms	211
	12.3 Intersection multiplicities	215
	12.4 Bezout Theorem	219
	12.5 Interpolation problems revisited	225
	12.6 Classification of projective varieties	229
	12.7 Exercises	231
	<b>Appendix A Notions from abstract algebra</b>	235
	A.1 Rings and homomorphisms	235
	A.2 Constructing new rings from old	236
	A.3 Modules	238
	A.4 Prime and maximal ideals	239
	A.5 Factorization of polynomials	240
	A.6 Field extensions	242
	A.7 Exercises	244
	<i>Bibliography</i>	246
	<i>Index</i>	249