## Lecture Notes in Mathematics

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## Ample Subvarieties of Algebraic Varieties

Notes written in Collaboration with C. Musili



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