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Combinatorics and Graph Theory

With 124 Illustrations

Springer

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To
Priscilla, Sophie,
Holly,
Kristine, and Amanda

Preface

Three things should be considered: problems, theorems, and applications.
— Gottfried Wilhelm Leibniz,
Dissertatio de Arte Combinatoria, 1666

This book grew out of several courses in combinatorics and graph theory given at Appalachian State University and UCLA in recent years. A one-semester course for juniors at Appalachian State University focusing on graph theory covered most of Chapter 1 and the first part of Chapter 2. A one-quarter course at UCLA on combinatorics for undergraduates concentrated on the topics in Chapter 2 and included some parts of Chapter 1. Another semester course at Appalachian State for advanced undergraduates and beginning graduate students covered most of the topics from all three chapters.

There are rather few prerequisites for this text. We assume some familiarity with basic proof techniques, like induction. A few topics in Chapter 1 assume some prior exposure to elementary linear algebra. Chapter 2 assumes some familiarity with sequences and series, especially Maclaurin series, at the level typically covered in a first-year calculus course. The text requires no prior experience with more advanced subjects, such as group theory.

While this book is primarily intended for upper-division undergraduate students, we believe that others will find it useful as well. Lower-division undergraduates with a penchant for proofs, and even talented high school students, will be able to follow much of the material, and graduate students looking for an introduction to topics in graph theory, combinatorics, and set theory may find several topics of interest.

Chapter 1 focuses on the theory of finite graphs. The first section serves as an introduction to basic terminology and concepts. Each of the following sections presents a specific branch of graph theory: trees, planarity, coloring, matchings, and Ramsey theory. These five topics were chosen for two reasons. First, they represent a broad range of the subfields of graph theory, and in turn they provide the reader with a sound introduction to the subject. Second, and just as important, these topics relate particularly well to topics in Chapters 2 and 3.

Chapter 2 develops the central techniques of enumerative combinatorics: the principle of inclusion and exclusion, the theory and application of generating functions, the solution of recurrence relations, Pólya's theory of counting arrangements in the presence of symmetry, and important classes of numbers, including the Fibonacci, Catalan, Stirling, Bell, and Eulerian numbers. The final section in the chapter continues the theme of matchings begun in Chapter 1 with a consideration of the stable marriage problem and the Gale–Shapley algorithm for solving it.

Chapter 3 presents infinite pigeonhole principles, König's Lemma, Ramsey's Theorem, and their connections to set theory. The systems of distinct representatives of Chapter 1 reappear in infinite form, linked to the axiom of choice. Counting is recast as cardinal arithmetic, and a pigeonhole property for cardinals leads to discussions of incompleteness and large cardinals. The last sections connect large cardinals to finite combinatorics and describe supplementary material on computability.

Following Leibniz's advice, we focus on problems, theorems, and applications throughout the text. We supply proofs of almost every theorem presented. We try to introduce each topic with an application or a concrete interpretation, and we often introduce more applications in the exercises at the end of each section. In addition, we believe that mathematics is a fun and lively subject, so we have tried to enliven our presentation with an occasional joke or (we hope) interesting quotation.

We would like to thank the Department of Mathematical Sciences at Appalachian State University and the Department of Mathematics at UCLA. We would especially like to thank our students (in particular, Jae-Il Shin at UCLA), whose questions and comments on preliminary versions of this text helped us to improve it. We would also like to thank the three anonymous reviewers, whose suggestions helped to shape this book into its present form. We also thank Sharon McPeake, a student at ASU, for her rendering of the Königsberg bridges.

In addition, the first author would like to thank Ron Gould, his graduate advisor at Emory University, for teaching him the methods and the joys of studying graphs, and for continuing to be his advisor even after graduation. He especially wants to thank his wife, Priscilla, for being his perfect match, and his daughter Sophie for adding color and brightness to each and every day. Their patience and support throughout this process have been immeasurable.

The second author would like to thank Judith Roitman, who introduced him to set theory and Ramsey's Theorem at the University of Kansas, using an early draft of her fine text. Also, he would like to thank his wife, Holly (the other Professor Hirst), for having the infinite tolerance that sets her apart from the norm.

The third author would like to thank Bob Blakley, from whom he first learned about combinatorics as an undergraduate at Texas A & M University, and Donald Knuth, whose class *Concrete Mathematics* at Stanford University taught him much more about the subject. Most of all, he would like to thank his wife, Kristine, for her constant support and infinite patience throughout the gestation of this project, and for being someone he can always, well, count on.

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