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Joe Harris

Algebraic Geometry

A First Course

With 83 Illustrations

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For Diane, Liam, and Davey

Preface

This book is based on one-semester courses given at Harvard in 1984, at Brown in 1985, and at Harvard in 1988. It is intended to be, as the title suggests, a first introduction to the subject. Even so, a few words are in order about the purposes of the book.

Algebraic geometry has developed tremendously over the last century. During the 19th century, the subject was practiced on a relatively concrete, down-to-earth level; the main objects of study were projective varieties, and the techniques for the most part were grounded in geometric constructions. This approach flourished during the middle of the century and reached its culmination in the work of the Italian school around the end of the 19th and the beginning of the 20th centuries. Ultimately, the subject was pushed beyond the limits of its foundations: by the end of its period the Italian school had progressed to the point where the language and techniques of the subject could no longer serve to express or carry out the ideas of its best practitioners.

This was more than amply remedied in the course of several developments beginning early in this century. To begin with, there was the pioneering work of Zariski who, aided by the German school of abstract algebraists, succeeded in putting the subject on a firm algebraic foundation. Around the same time, Weil introduced the notion of abstract algebraic variety, in effect redefining the basic objects studied in the subject. Then in the 1950s came Serre's work, introducing the fundamental tool of sheaf theory. Finally (for now), in the 1960s, Grothendieck (aided and abetted by Artin, Mumford, and many others) introduced the concept of the scheme. This, more than anything else, transformed the subject, putting it on a radically new footing. As a result of these various developments much of the more advanced work of the Italian school could be put on a solid foundation and carried further; this has been happening over the last two decades simultaneously with the advent of new ideas made possible by the modern theory.

All this means that people studying algebraic geometry today are in the position of being given tools of remarkable power. At the same time, didactically it creates a dilemma: what is the best way to go about learning the subject? If your goal is simply to see what algebraic geometry is about—to get a sense of the basic objects considered, the questions asked about them and the sort of answers one can obtain—you might not want to start off with the more technical side of the subject. If, on the other hand, your ultimate goal is to work in the field of algebraic geometry it might seem that the best thing to do is to introduce the modern approach early on and develop the whole subject in these terms. Even in this case, though, you might be better motivated to learn the language of schemes, and better able to appreciate the insights offered by it, if you had some acquaintance with elementary algebraic geometry.

In the end, it is the subject itself that decided the issue for me. Classical algebraic geometry is simply a glorious subject, one with a beautifully intricate structure and yet a tremendous wealth of examples. It is full of enticing and easily posed problems, ranging from the tractable to the still unsolved. It is, in short, a joy both to teach and to learn. For all these reasons, it seemed to me that the best way to approach the subject is to spend some time introducing elementary algebraic geometry before going on to the modern theory. This book represents my attempt at such an introduction.

This motivation underlies many of the choices made in the contents of the book. For one thing, given that those who want to go on in algebraic geometry will be relearning the foundations in the modern language there is no point in introducing at this stage more than an absolute minimum of technical machinery. Likewise, I have for the most part avoided topics that I felt could be better dealt with from a more advanced perspective, focussing instead on those that to my mind are nearly as well understood classically as they are in modern language. (This is not absolute, of course; the reader who is familiar with the theory of schemes will find lots of places where we would all be much happier if I could just say the words "scheme-theoretic intersection" or "flat family".)

This decision as to content and level in turn influences a number of other questions of organization and style. For example, it seemed a good idea for the present purposes to stress examples throughout, with the theory developed concurrently as needed. Thus, Part I is concerned with introducing basic varieties and constructions; many fundamental notions such as dimension and degree are not formally defined until Part II. Likewise, there are a number of unproved assertions, theorems whose statements I thought might be illuminating, but whose proofs are beyond the scope of the techniques introduced here. Finally, I have tried to maintain an informal style throughout.

Acknowledgments

Many people have helped a great deal in the development of this manuscript. Benji Fisher, as a junior at Harvard, went to the course the first time it was given and took a wonderful set of notes; it was the quality of those notes that encouraged me to proceed with the book. Those who attended those courses provided many ideas, suggestions, and corrections, as did a number of people who read various versions of the book, including Paolo Aluffi, Dan Grayson, Zinovy Reichstein and John Tate. I have also enjoyed and benefited from conversations with many people including Fernando Cukierman, David Eisenbud, Noam Elkies, Rolfdieter Frank, Bill Fulton, Dick Gross and Kurt Mederer.

The references in this book are scant, and I apologize to those whose work I may have failed to cite properly. I have acquired much of my knowledge of this subject informally, and remain much less familiar with the literature than I should be. Certainly, the absence of a reference for any particular discussion should be taken simply as an indication of my ignorance in this regard, rather than as a claim of originality.

I would like to thank Harvard University, and in particular Deans Candace Corvey and A. Michael Spence, for their generosity in providing the computers on which this book was written.

Finally, two people in particular contributed enormously and deserve special mention. Bill Fulton and David Eisenbud read the next-to-final version of the manuscript with exceptional thoroughness and made extremely valuable comments on everything from typos to issues of mathematical completeness and accuracy. Moreover, in every case where they saw an issue, they proposed ways of dealing with it, most of which were far superior to those I could have come up with.

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Using This Book

There is not much to say here, but I'll make a couple of obvious points.

First of all, a quick glance at the book will show that the logical skeleton of this book occupies relatively little of its volume: most of the bulk is taken up by examples and exercises. Most of these can be omitted, if they are not of interest, and gone back to later if desired. Indeed, while I clearly feel that these sorts of examples represent a good way to become familiar with the subject, I expect that only someone who was truly gluttonous, masochistic, or compulsive would read every single one on the first go-round. By way of example, one possible abbreviated tour of the book might omit (hyphens without numbers following mean "to end of lecture") 1.22–, 2.27–, 3.16–, 4.10–, 5.11–, 6.8–11, 7.19–21, 7.25–, 8.9–13, 8.32–39, 9.15–20, 10.12–17, 10.23–, 11.40–, 12.11–, 13.7–, 15.7–21, 16.9–11, 16.21–, 17.4–15, 19.11–, 20.4–6, 20.9–13 and all of 21.

By the same token, I would encourage the reader to jump around in the text. As noted, some basic topics are relegated to later in the book, but there is no reason not to go ahead and look at these lectures if you're curious. Likewise, most of the examples are dealt with several times: they are introduced early and reexamined in the light of each new development. If you would rather, you could use the index and follow each one through.

Lastly, a word about prerequisites (and post-requisites). I have tried to keep the former to a minimum: a reader should be able to get by with just some linear and multilinear algebra and a basic background in abstract algebra (definitions and basic properties of groups, rings, fields, etc.), especially with a copy of a user-friendly commutative algebra book such as Atiyah and MacDonald's [AM] or Eisenbud's [E] at hand.

At the other end, what to do if, after reading this book, you would like to learn some algebraic geometry? The next step would be to learn some sheaf theory, sheaf cohomology, and scheme theory (the latter two not necessarily in that order).

For sheaf theory in the context of algebraic geometry, Serre's paper [S] is the basic source. For the theory of schemes, Hartshorne's [H] classic book stands out as the canonical reference; as an introduction to the subject there is also Mumford's [M1] red book and the book by Eisenbud and Harris [EH]. Alternatively, for a discussion of some advanced topics in the setting of complex manifolds rather than schemes, see [GH].

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