

Harmonic Analysis and Boundary Value Problems in the Complex Domain

Mkhitar M. Djrbashian

Springer Basel AG

Author

Mkhitar M. Djrbashian
Academy of Sciences of Armenia
Institute of Mathematics
Marshal Bagramian Ave. 24-B
Yerevan, 375019
Armenia

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA

Deutsche Bibliothek Cataloging-in-Publication Data

Djrbashian, Mkhitar M.:

Harmonic analysis and boundary value problems in the complex
domain / Mkhitar M. Djrbashian. - Basel ; Boston ; Berlin
Birkhäuser, 1993

(Operator theory ; Vol. 65)

ISBN 978-3-0348-9674-0 ISBN 978-3-0348-8549-2 (eBook)

DOI 10.1007/978-3-0348-8549-2

NE: GT

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use the permission of the copyright holder must be obtained.

© 1993 Springer Basel AG

Originally published by Birkhäuser Verlag in 1993

Softcover reprint of the hardcover 1st edition 1993

Printed on acid-free paper produced from chlorine-free pulp

Cover design: Heinz Hiltbrunner, Basel

ISBN 978-3-0348-9674-0

9 8 7 6 5 4 3 2 1

Table of Contents

Preface	xi
1 Preliminary results.	
Integral transforms in the complex domain	
1.1 Introduction	1
1.2 Some identities	1
1.3 Integral representations and asymptotic formulas	3
1.4 Distribution of zeros	6
1.5 Identities between some Mellin transforms	10
1.6 Fourier type transforms with Mittag-Leffler kernels	12
1.7 Some consequences	17
1.8 Notes	21
2 Further results. Wiener-Paley type theorems	
2.1 Introduction	23
2.2 Some simple generalizations of the first fundamental Wiener-Paley theorem	23
2.3 A general Wiener-Paley type theorem and some particular results	26
2.4 Two important cases of the general Wiener-Paley type theorem	29
2.5 Generalizations of the second fundamental Wiener-Paley theorem	31
2.6 Notes	34
3 Some estimates in Banach spaces of analytic functions	
3.1 Introduction	35
3.2 Some estimates in Hardy classes over a half-plane	35
3.3 Some estimates in weighted Hardy classes over a half-plane	43
3.4 Some estimates in Banach spaces of entire functions of exponential type	49
3.5 Notes	59
4 Interpolation series expansions in spaces $W_{1/2,\sigma}^{p,\omega}$ of entire functions	
4.1 Introduction	61
4.2 Lemmas on special Mittag-Leffler type functions	61
4.3 Two special interpolation series	70
4.4 Interpolation series expansions	78
4.5 Notes	82

5	Fourier type basic systems in $L_2(0, \sigma)$	
5.1	Introduction	83
5.2	Biorthogonal systems of Mittag-Leffler type functions and their completeness in $L_2(0, \sigma)$	83
5.3	Fourier series type biorthogonal expansions in $L_2(0, \sigma)$	89
5.4	Notes	93
6	Interpolation series expansions in spaces $W_{s+1/2, \sigma}^{p, \omega}$ of entire functions	
6.1	Introduction	95
6.2	The formulation of the main theorems	95
6.3	Auxiliary relations and lemmas	99
6.4	Further auxiliary results	106
6.5	Proofs of the main theorems	113
6.6	Notes	120
7	Basic Fourier type systems in L_2 spaces of odd-dimensional vector functions	
7.1	Introduction	121
7.2	Some identities	121
7.3	Biorthogonal systems of odd-dimensional vector functions	126
7.4	Theorems on completeness and basis property	132
7.5	Notes	140
8	Interpolation series expansions in spaces $W_{s, \sigma}^{p, \omega}$ of entire functions	
8.1	Introduction	141
8.2	The formulation of the main interpolation theorem	141
8.3	Auxiliary relations and lemmas	147
8.4	Further auxiliary results	153
8.5	The proof of the main interpolation theorem	159
8.6	Notes	163
9	Basic Fourier type systems in L_2 spaces of even-dimensional vector functions	
9.1	Introduction	165
9.2	Some identities	165
9.3	The construction of biorthogonal systems of even-dimensional vector functions	168
9.4	Theorems on completeness and basis property	171
9.5	Notes	176

10	The simplest Cauchy type problems and the boundary value problems connected with them	
10.1	Introduction	177
10.2	Riemann-Liouville fractional integrals and derivatives	178
10.3	A Cauchy type problem	188
10.4	The associated Cauchy type problem and the analog of Lagrange formula	195
10.5	Boundary value problems and eigenfunction expansions	202
10.6	Notes	210
11	Cauchy type problems and boundary value problems in the complex domain (the case of odd segments)	
11.1	Introduction	211
11.2	Preliminaries	211
11.3	Cauchy type problems and boundary value problems containing the operators $\mathbb{L}_{s+1/2}$ and $\mathbb{L}_{s+1/2}^*$	213
11.4	Expansions in $L_2\{\gamma_{2s+1}(\sigma)\}$ in terms of Riesz bases	223
11.5	Notes	233
12	Cauchy type problems and boundary value problems in the complex domain (the case of even segments)	
12.1	Introduction	235
12.2	Preliminaries	235
12.3	Cauchy type problems and boundary value problems containing the operators $\mathbb{L}_s$ and $\mathbb{L}_s^*$	237
12.4	Expansions in $L_2\{\gamma_{2s}(\sigma)\}$ in terms of Riesz bases	244
12.5	Notes	251
	Bibliography	253