

Collected Papers of
SRINIVASA RAMANUJAN

Edited by

G. H. HARDY
P. V. SESHU AIYAR
and
B. M. WILSON

AMS CHELSEA PUBLISHING

American Mathematical Society • Providence, Rhode Island

CONTENTS

	PAGE
PREFACE	ix
NOTICE by P. V. SESHU AIYAR and R. BAMACHAUNDR A RAO	xi
NOTICE by G. H. HARDY	xxi
PREFACE TO THE THIRD PRINTING by BRUCE C. BERNDT <i>(1999)</i>	xxxvii
 PAPERS :	
1. Some properties of Bernoulli's numbers	1
<i>Journal of the Indian Mathematical Society</i> , 3 (1911), 219–234.	
2. On Question 330 of Prof. Sanjana	15
<i>Journal of the Indian Mathematical Society</i> , 4 (1912), 59–61.	
3. Note on a set of simultaneous equations	18
<i>Journal of the Indian Mathematical Society</i> , 4 (1912), 94–96.	
4. Irregular numbers	20
<i>Journal of the Indian Mathematical Society</i> , 5 (1913), 105–106.	
5. Squaring the circle	22
<i>Journal of the Indian Mathematical Society</i> , 5 (1913), 132.	
6. Modular equations and approximations to π	23
<i>Quarterly Journal of Mathematics</i> , 45 (1914), 350–372.	
7. On the integral $\int_0^x \frac{\tan^{-1} t}{t} dt$	40
<i>Journal of the Indian Mathematical Society</i> , 7 (1915), 93–96.	
8. On the number of divisors of a number	44
<i>Journal of the Indian Mathematical Society</i> , 7 (1915), 131–133.	
9. On the sum of the square roots of the first n natural numbers	47
<i>Journal of the Indian Mathematical Society</i> , 7 (1915), 173–175.	
10. On the product $\prod_{n=0}^{n=\infty} \left[1 + \left(\frac{x}{a+nd} \right)^3 \right]$	50
<i>Journal of the Indian Mathematical Society</i> , 7 (1915), 209–211.	
11. Some definite integrals	53
<i>Messenger of Mathematics</i> , 44 (1915), 10–18.	
12. Some definite integrals connected with Gauss's sums	59
<i>Messenger of Mathematics</i> , 44 (1915), 75–85.	
13. Summation of a certain series	68
<i>Messenger of Mathematics</i> , 44 (1915), 157–160.	
14. New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$	72
<i>Quarterly Journal of Mathematics</i> , 46 (1915), 253–260.	

	PAGE
15. Highly composite numbers <i>Proceedings of the London Mathematical Society</i> , 2, 14 (1915), 347-409.	78
16. On certain infinite series <i>Messenger of Mathematics</i> , 45 (1916), 11-15.	129
17. Some formulæ in the analytic theory of numbers <i>Messenger of Mathematics</i> , 45 (1916), 81-84.	133
18. On certain arithmetical functions <i>Transactions of the Cambridge Philosophical Society</i> , 22, No. 9 (1916), 159-184.	136
19. A series for Euler's constant γ <i>Messenger of Mathematics</i> , 46 (1917), 73-80.	163
20. On the expression of a number in the form $ax^3 + by^3 + cz^3 + du^3$. <i>Proceedings of the Cambridge Philosophical Society</i> , 19 (1917), 11-21.	169
21. On certain trigonometrical sums and their applications in the theory of numbers <i>Transactions of the Cambridge Philosophical Society</i> , 22, No. 13 (1918), 259-276.	179
22. Some definite integrals <i>Proceedings of the London Mathematical Society</i> , 2, 17 (1918), <i>Records for 17 Jan. 1918</i> .	200
23. Some definite integrals <i>Journal of the Indian Mathematical Society</i> , 11 (1919), 81-87.	202
24. A proof of Bertrand's postulate <i>Journal of the Indian Mathematical Society</i> , 11 (1919), 181-182.	208
25. Some properties of $p(n)$, the number of partitions of n <i>Proceedings of the Cambridge Philosophical Society</i> , 19 (1919), 207-210.	210
26. Proof of certain identities in combinatory analysis <i>Proceedings of the Cambridge Philosophical Society</i> , 19 (1919), 214-216.	214
27. A class of definite integrals <i>Quarterly Journal of Mathematics</i> , 48 (1920), 294-310.	216
28. Congruence properties of partitions <i>Proceedings of the London Mathematical Society</i> , 2, 18 (1920), <i>Records for 13 March 1919</i> .	230
29. Algebraic relations between certain infinite products <i>Proceedings of the London Mathematical Society</i> , 2, 18 (1920), <i>Records for 13 March 1919</i> .	231
30. Congruence properties of partitions <i>Mathematische Zeitschrift</i> , 9 (1921), 147-153.	232

Contents

vii

PAPERS written in collaboration with G. H. HARDY:	PAGE
31. Une formule asymptotique pour le nombre des partitions de n . <i>Comptes Rendus</i> , 2 Jan. 1917.	239
32. Proof that almost all numbers n are composed of about $\log \log n$ prime factors <i>Proceedings of the London Mathematical Society</i> , 2, 16 (1917), <i>Records for 14 Dec. 1916.</i>	242
33. Asymptotic formulæ in combinatory analysis <i>Proceedings of the London Mathematical Society</i> , 2, 16 (1917), <i>Records for 1 March 1917.</i>	244
34. Asymptotic formulæ for the distribution of integers of various types <i>Proceedings of the London Mathematical Society</i> , 2, 16 (1917), 112–132.	245
35. The normal number of prime factors of a number n <i>Quarterly Journal of Mathematics</i> , 48 (1917), 76–92.	262
36. Asymptotic formulæ in combinatory analysis <i>Proceedings of the London Mathematical Society</i> , 2, 17 (1918), 75–115.	276
37. On the coefficients in the expansions of certain modular functions <i>Proceedings of the Royal Society, A</i> , 95 (1918), 144–155.	310
QUESTIONS AND SOLUTIONS	322
APPENDIX I: NOTES ON THE PAPERS	335
APPENDIX II: FURTHER EXTRACTS FROM RAMANUJAN'S LETTERS TO G. H. HARDY	349
COMMENTARY ON RAMANUJAN'S COLLECTED PAPERS by Bruce C. Berndt	357