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Numerical Mathematics

Translated by Larry Schumaker

With 76 Illustrations

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Preface

> "In truth, it is not knowledge, but learning, not possessing, but production, not being there, but travelling there, which provides the greatest pleasure. When I have completely understood something, then I turn away and move on into the dark; indeed, so curious is the insatiable man, that when he has completed one house, rather than living in it peacefully, he starts to build another."
>
> Letter from C. F. Gauss to W. Bolyai on Sept. 2, 1808

This textbook adds a book devoted to applied mathematics to the series "Grundwissen Mathematik." Our goals, like those of the other books in the series, are to explain connections and common viewpoints between various mathematical areas, to emphasize the motivation for studying certain problem areas, and to present the historical development of our subject.

Our aim in this book is to discuss some of the central problems which arise in applications of mathematics, to develop constructive methods for the numerical solution of these problems, and to study the associated questions of accuracy. In doing so, we also present some theoretical results needed for our development, especially when they involve material which is beyond the scope of the usual beginning courses in calculus and linear algebra. This book is based on lectures given over many years at the Universities of Freiburg, Munich, Berlin and Augsburg. Our intention is not simply to give a set of recipes for solving problems, but rather to present the underlying mathematical structure. In this sense, we agree with R. W. Hamming [1962] that the purpose of numerical analysis is "insight, not numbers."

In choosing material to include here, our main criterion was that it should show how one typically approaches problems in numerical analysis. In addition, we have tried to make the book sufficiently complete so as to provide a solid basis for studying more specialized areas of numerical analysis, such as the solution of differential or integral equations, nonlinear optimization, or integral transforms. Thus, cross-connections and open questions have also been discussed. In summary, we have tried to select material and to organize it in such a way as to meet our mathematical goals, while at the same time giving the reader some of the feeling of joy that Gauss expressed in his letter quoted at the beginning of this preface.

The amount of material in this book exceeds what is usually covered in a two semester course. Thus, the instructor has a variety of possibilities for selecting material. If you are a student who is using this book as a supplement to other course materials, we hope that our presentation covers all of the material contained in your course, and that it will help deepen your understanding and provide new insights. Chapter 1 of the book deals with the basic question of arithmetic, and in particular how it is done by machines. We start the book with this subject since all of mathematics grows out of numbers, and since numerical analysis must deal with them. However, it is not absolutely necessary to study Chapter 1 in detail before proceeding to the following chapters. The remaining chapters can be divided into two major parts: Chapters 4 – 7 along with Sections 1 and 2 of Chapter 8 deal with classical problems of numerical analysis. Chapters 2, 3 and 9, and the remainder of Chapter 8 are devoted to numerical linear algebra.

A number of our colleagues were involved in the development and production of this book. We thank all of them heartily. In particular, we would like to mention L. Bamberger, A. Burgstaller, P. Knabner, M. Hilpert, E. Schäfer, U. Schmid, D. Schuster, W. Spann and M. Thoma for suggestions, for reading parts of the manuscript and galley proofs, and for putting together the index. We would like to thank I. Eichenseher for mastering the mysteries of TEX; C. Niederauer and K. Bernt for preparing the figures and tables; and H. Hornung and I. Mignani for typing parts of the manuscript. Our special thanks are due to M.-E. Eberle for her skillful preparation of the camera-ready copy of the book, and her patient willingness to go through many revision with the authors.

Munich and Augsburg G. Hämmerlin

December, 1988 K.-H. Hoffmann

Note to the reader: This book contains a total of 270 exercises of various degrees of difficulty. These can be found at the end of each section. Cross references to material in other sections or subsections of a given chapter will be made by referring only to the section and subsection number. Otherwise, the chapter number is placed in front of them. We use square brackets [·] to refer to the papers and books listed at the end.

Translator's Note: This book is a direct translation of the first German edition, with only very minor changes. Several misprints have been corrected, and some English language references have been added or substituted for the original German ones. I would like to thank my wife, Gerda, for her help in preparing the translation and the camera-ready manuscript.

Munich, July, 1990 L. L. Schumaker

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