# Applied Mathematical Sciences
Volume 99

# Applied Mathematical Sciences

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# Introduction to Functional Differential Equations

With 10 Illustrations

Springer Science+Business Media, LLC

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Mathematics Subject Classification (1991): 34K20, 34A30, 39A10

Library of Congress Cataloging-in-Publication Data
Hale, Jack K.
Introduction to functional differential equations/Jack K. Hale, Sjoerd M. Verduyn Lunel.
p. cm. — (Applied mathematical sciences ; v.)
Includes bibliographical references.
ISBN 978-1-4612-8741-4 ISBN 978-1-4612-4342-7 (eBook)
DOI 10.1007/978-1-4612-4342-7
1. Functional differential equations. I. Verduyn Lunel, S. M. (Sjoerd M.) II. Title. III. Series: Applied mathematical sciences (Springer-Verlag New York Inc.) ; v.
QA1.A647
[QA372]
510 s—dc20
[515′.35] 93-27729

Printed on acid-free paper.



Production managed by Natalie Johnson; manufacturing supervised by Jacqui Ashri.
Photocomposed copy produced using the authors' TeX files.

9 8 7 6 5 4 3 2 1

# Preface

The present book builds upon an earlier work of J. Hale, "Theory of Functional Differential Equations" published in 1977. We have tried to maintain the spirit of that book and have retained approximately one-third of the material intact. One major change was a complete new presentation of linear systems (Chapters 6–9) for retarded and neutral functional differential equations. The theory of dissipative systems (Chapter 4) and global attractors was completely revamped as well as the invariant manifold theory (Chapter 10) near equilibrium points and periodic orbits. A more complete theory of neutral equations is presented (see Chapters 1, 2, 3, 9, and 10). Chapter 12 is completely new and contains a guide to active topics of research. In the sections on supplementary remarks, we have included many references to recent literature, but, of course, not nearly all, because the subject is so extensive.

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