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Dynamics in Infinite Dimensions

Second Edition

Appendix by Krzysztof P. Rybakowski

With 15 Figures



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