

Hermann Haken

# Brain Dynamics

Synchronization and Activity Patterns  
in Pulse-Coupled Neural Nets  
with Delays and Noise

With 82 Figures

 Springer

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