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The Classical Groups and K-Theory

Foreword by J. Dieudonné

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Alexander J. Hahn
Department of Mathematics
University of Notre Dame
Notre Dame, IN 46556, USA

O. Timothy O'Meara
Provost
University of Notre Dame
Notre Dame, IN 46556, USA

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To
Marianne and Jean

Foreword

It is a great satisfaction for a mathematician to witness the growth and expansion of a theory in which he has taken some part during its early years.

When H. Weyl coined the words "classical groups", foremost in his mind were their connections with invariant theory, which his famous book helped to revive. Although his approach in that book was deliberately algebraic, his interest in these groups directly derived from his pioneering study of the special case in which the scalars are real or complex numbers, where for the first time he injected Topology into Lie theory. But ever since the definition of Lie groups, the analogy between simple classical groups over finite fields and simple classical groups over $\mathbb{R}$ or $\mathbb{C}$ had been observed, even if the concept of "simplicity" was not quite the same in both cases. With the discovery of the exceptional simple complex Lie algebras by Killing and E. Cartan, it was natural to look for corresponding groups over finite fields, and already around 1900 this was done by Dickson for the exceptional Lie algebras G_2 and E_6. However, a deep reason for this parallelism was missing, and it is only Chevalley who, in 1955 and 1961, discovered that to each complex simple Lie algebra corresponds, by a *uniform* process, a group scheme $\mathfrak{G}$ over the ring $\mathbb{Z}$ of integers, from which, for *any* field K, could be derived a group $\mathfrak{G}(K)$. Furthermore, the Chevalley construction provided a *general* proof for all the "simplicity" theorems, obtained until then by *ad hoc* methods in each particular case.

Classical groups can be defined when the scalars only form a ring (commutative in most cases). The methods used in their study when the scalars form a field can be slightly extended to local rings; but for more general rings, they don't apply any more, and new ideas were needed. They were brilliantly provided by O'Meara; he grouped around him at Notre Dame a school of younger mathematicians who developed his methods in several directions, and elucidated many properties of the structure of classical groups over rings and of their isomorphisms. More recently, unexpected connections of classical groups with K-theory have been discovered; one of the most active participants in their development has been A. Hahn. All mathematicians interested in classical groups should be grateful to these two outstanding investigators for having brought together old and new results (many of them their own) into a superbly organized whole. I am confident that their book will remain for a long time the standard reference in the theory.

J. Dieudonné

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March 21, 1989 A. J. Hahn and O. T. O'Meara

Table of Contents