Hubert Hahn

Rigid Body Dynamics of Mechanisms 1

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1 Theoretical Basis

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To
Mechthild and Elke

Preface

The dynamics of mechanical rigid-body mechanisms is a highly developed discipline. The model equations that apply to the tremendous variety of applications of rigid-body systems in industrial practice are based on just a few basic laws of, for example, Newton, Euler, or Lagrange. These basic laws can be written in an extremely compact, symmetrical, and esthetic form, simple enough to be easily learned and kept in mind by students and engineers, not only from the area of mechanics but also from other disciplines such as physics, or mathematics, or even control, hydraulics, or electronics. This latter aspect is of immense practical importance since mechanisms, machines, robots, and vehicles in modern industrial practice (sometimes called mechatronic systems) usually include various subsystems from the areas of hydraulics, electronics, pneumatics, informatics, and control, and are built by engineers trained in quite different disciplines.

Conventional methods of modeling rigid-body mechanisms

In contrast to the comparatively simple and easy-to-learn basic laws of rigid-body systems, the practical application of these laws to the planar or spatial motions of industrial mechanisms rapidly leads to extremely lengthy and complex equations of motion, where the form and complexity of the model equations depends critically on the choice of the model coordinates. Until recently this had the following consequences:

1. A large variety of *specialized techniques* have been developed, each suitable for efficiently modeling a *special-purpose mechanism.*
2. These techniques have usually been applied to comparatively *simple mechanisms*, as most of them were developed at universities or academic institutes, where there was no need to model complex realistic industrial systems, and no pressure to do this within a predetermined time schedule.
3. The overwhelming *majority of practicing industrial engineers have not had the opportunity to learn all these special modeling techniques.* They were usually neither capable of finding a special modeling approach suitable to a given mechanism, nor of deriving efficiently and correctly the realistic model equations, nor of estimating in advance the effort required to derive those models and to set up a time schedule for the task.

As a consequence there has been a large gap between the available basic laws of mechanics and the ability of practicing industrial engineers to apply them to large rigid-body systems.

General-purpose rigid-body analysis programs as efficient modeling tools

In the past two decades the above problems have been overcome by worldwide intensive research activity. As a result, various *general-purpose rigid-body analysis programs* have been developed that:

1. *Automatically set up the equations of motion* of rather complex kinematic and dynamic mechanisms.
2. *Provide efficient and accurate computer simulations* of most of these systems.
3. *Perform the first analysis steps*, such as static analysis, kinematic analysis, local linearization, eigenvalue analysis, and sensitivity analysis.

Examples of general-purpose rigid-body analysis programs include ADAMS ([1],[2]), DADS ([3]), NUSTAR ([4], [5]), and various other software packages discussed in ([6], [7]). Teaching computers to automatically formulating the equations of motion was equivalent to developing *systematic general methods* for *setting up* and *solving model equations of quite general mechanisms.* Using these computer programs, practicing industrial engineers can simulate and analyse complex rigid-body systems:

1. By *setting up an engineering model* of the mechanism based on their intuitive practical understanding of that system.
2. By *handling a rigid-body analysis program* without the burden of deriving complex analytical model equations, developing computer simulation code, and developing numerical solution algorithms of these equations.

Many of these rigid-body analysis programs have been equipped with graphical user interfaces that can be easily handled even by engineers who have a limited understanding both of the underlying mechanics and numerics, and of the *problems that may occur in the computer-aided modeling and solution process.* However this latter inexperience may have serious consequences: numerical results may be obtained by these programs that are far more erroneous than any results obtained in laboratory experiments.

Objectives of this monograph

Volume I of this monograph presents:

1. An introduction into the *theoretical background* of rigid-body mechanics.
2. A *systematic approach* for *deriving model equations* of mechanisms, as a first step in symbolic *differential–algebraic equations (DAE) form.*

Volume II presents:

1. Various *exercises to systematically apply this approach to examples* of *planar and spatial mechanisms.*
2. A symbolic approach for mapping the DAEs in a second step into symbolic *differential equations (DEs)*, into *nonlinear* and *linear state-space equations*, and sometimes also into *transfer function form.*

The *objectives* of both the *theoretical discussions* (Volume I) and the *practical applications* (Volume II) are:

1. *To prepare the reader for efficiently handling and application of general-purpose rigid-body analysis programs to complex mechanisms*, and
2. To *set up symbolic mathematical models of mechanisms in DAE form* for *computer simulations* and/or in *DE form*, as is often required in *dynamic analysis* and *control design.*

From the point of view of these two objectives this monograph can be considered as an introduction to basic mechanical aspects of *mechatronic systems.*

Organization of the books (Volumes I and II)

The two volumes of this monograph provide a *systematic theoretical approach for setting up model equations of planar and spatial rigid-body systems* in *DAE form* (Volume I), and present various *applications of the modeling methodology* to examples of planar and spatial mechanisms (Volume II).

Volume I includes *six chapters* and *four appendices. Chapter 1* gives a brief introduction to the subject of modeling rigid-body mechanisms, which is illustrated by several simple examples and by some more complex applications of mechanisms from industrial practice. *Chapter 2* presents a brief review of vector and matrix algebra and of multivariable calculus for the planar and spatial cases. Spatial rotations are derived in terms of Bryant angles together with the associated kinematic DEs. Due to the introductory character of this book, quaternions or Euler parameters of spatial rotations are not considered here (despite the fact that singularities may occur in the kinematic DEs of Bryant angles). Time derivatives of vector functions together with the gradient vector and the Jacobian matrix of those functions are introduced. They will be used extensively for describing constraint relations. Some useful relations of scalar products and cross products of vectors are derived in *Appendix A.1*, together with different expressions for the time derivatives of vectors and orientation matrices of planar and spatial vectors, and with a brief review of derivatives of vector functions. Relations of planar and spatial kinematic and active constraints, represented in Cartesian coordinates, are discussed in *Chapter 3* together with the associated velocity and acceleration constraint equations, including formal relationships between constraint reaction forces and torques, and with a discussion of possible singularities of the constraint equations, illustrated by an example. Kinetic equations of planar and spatial rigid-body mechanisms are developed in *Chapter 4* and in *Appendix A.2*. Starting with the concepts of linear momentum and angular momentum in *Section 4.1*, the Newton–Euler equations of the planar and spatial motion of a single unconstrained rigid body are derived in *Section 4.2*, together with the model equations of planar and spatial mechanisms in *Section 4.3*. A brief discussion of the numerical solution of DAEs is presented in *Section 4.4*. Parallel to the Newton–Euler approach, the Lagrange formalism is briefly discussed in *Appendix A.2*. Basic differences between the theoretical constituents of planar and spatial mechanisms are collected in *Appendix A.3*. In *Chapter 5* a systematic approach for deriving the constraint equations of planar and spatial joints is presented based on suitable representations and projections of vector and orientation loop equations. The constraint equations of various joint types in common use are derived there. Theoretical models of joints of planar mechanisms are presented in *Section 5.1*. Model equations of joints of spatial mechanisms are derived in *Section 5.2* and in *Appendix A.4*. Constitutive relations of applied forces and torques of planar and spatial mechanisms are discussed in *Chapter 6*. Among those, theoretical models of translational and torsional springs and dampers as well as models of actuators and motors are briefly presented.

Various simple and some more complex applications of rigid-body mechanisms are modeled in symbolic *DAE form* and in *DE form*, and for selected mechanisms also in *nonlinear* and *linear state-space form* and using the *transfer function matrix* representation in *Volume II*. They include various combinations of theoretical models of joints, and of active and passive force elements. In *Chapter 1* of Volume II, the modeling methodology is summarized, and a software package is briefly discussed ([8]) that maps symbolic model equations from DAE form into DE form (in most cases where this is feasible). Two applications of *planar models* of an unconstrained rigid body are discussed in *Chapter 2*. Several applications of a planar rigid body under constrained motion are presented in *Chapter 3*. Various applications of planar mechanisms that include two rigid bodies under constraints are discussed in *Chapter 4*. Applications of a rigid body under unconstrained *spatial motion* are collected in *Chapter 5*, followed by several applications of a constrained spatial rigid body in *Chapter 6*, and by several applications of spatial mechanisms including between two and thirteen constrained rigid bodies in *Chapter 7*.

Use of the text

The text of the books is intended for use and self-study by practicing industrial engineers that have a bachelor's degree, and by students of undergraduate university courses. The contents of the books have been used in lectures and courses held over many years:

1. In several *industrial companies* (like BMW and IABG) for practicing engineers from the areas of mechanics, vibration techniques, vehicle simulation, control, hydraulics, pneumatics, measurement, testing, electromagnetics, and electronics.
2. In the undergraduate courses of several *universities* (Universities of Munich, Tübingen, and Kassel) for students from the areas of mechanical engineering, control engineering, electrical engineering, civil engineering, physics, and mathematics.

The practicing engineers who attended these courses have influenced both the contents and the direction of this monograph, resulting in more emphasis being placed on:

1. A *systematic choice of notation* (with indices of the variables that uniquely identify the frames of their representations and time derivatives).
2. An *algebraic formulation* of all expressions in a form suitable for direct *implementation in a computer.*
3. *Applying these methods to both simple and complex mechanisms.*

The engineers and students that attended these lectures had the opportunity to apply these methods to practical examples of mechanisms using general-purpose rigid-body analysis programs like NUSTAR, ADAMS, and DADS.

Spatial mechanics is conceptually more complex and its theoretical modeling provides much lengthier and more unwieldy formal expressions than *planar* mechanics. To enable the beginner reader to successfully master his or her study of rigid-body dynamics and to keep the amount of notation and formal expressions of the applications presented within acceptable limits, only *planar* rigid-body systems are considered in the first parts of *Chapters 2, 3, 5* and *6 of Volume I.* They present vectors, matrices, kinematics, forces and torques of *planar* geometry and *planar* mechanics. The equations of motion of rigid bodies under *planar* motion are collected in *Chapter 4 of Volume I.* Various *planar* mechanisms are discussed in *Chapters 2, 3* and *4 of Volume II.* Teaching experience shows that the methodology of modeling rigid-body systems can be basically understood by considering *planar* systems only. Having developed confidence and enough intuition in the basic methods of theoretical modeling of *planar* mechanisms, the reader is encouraged to study *spatial* mechanisms in the second parts of *Chapters 2, 3, 5, 6 and in all of Chapter 4 of Volume I*, and the applications of *spatial* mechanisms of *Chapters 5, 6*, and *7 of Volume II.* Basic differences between the model equations of *planar* and *spatial* mechanisms are summarized in *Appendix A.3* of *Volume I.*

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Contents