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Introduction

Soliton theory is an important subject in applied mathematics and mathematical physics, which has developed rapidly since the sixties.

A soliton or solitary wave occurs in the solution of various nonlinear partial differential equations; it has some striking properties and describes several important physical phenomena. In physical language, these properties are: (i) energy centered within a small region; (ii) an elastic scattering phenomenon in the interaction of two solitons (that is, the shape and velocity of the wave can recover after interaction). Solitons behave both as particles and as waves, and occur frequently in nature. In research field such as fluid mechanics, plasma physics, nonlinear optics, classical and quantum field theory etc., there are many important problems related to soliton theory. In recent years, the notion of a "soliton" has also become understood in a more general sense. For instance, static solutions with property (i) are sometimes called solitons.

The origin and development of soliton theory represent a great event in the study of nonlinear partial differential equations. As is well known, the Fourier method is very useful in many linear problems of mathematical physics. With the Fourier transformation, one can obtain exact solutions to the problem. For nonlinear partial differential equations, the situation is much more difficult. However, soliton theory provided many methods to treat nonlinear problems. The inverse scattering method in particular can be considered in some sense as the Fourier method for nonlinear problems. Besides the inverse scattering approach, there are plenty of elegant and efficient methods of constructing exact solutions. Many branches of mathematics, such as classical and functional analysis, Lie groups, Lie algebras, differential geometry, algebraic geometry, topology, dynamical systems and computational mathematics, provide important tools for the study of solitons. On the other hand, the study of solitons also promotes the development of these areas.

For these reasons, both mathematicians and physicists pay much attention to soliton theory. It is a very active research field, and covers an increasing range of subjects. In each of the last ten years, several international conferences on this field were held. Several books have been published, and there are many papers on soliton theory in various journals. There are research groups in this field in many countries, each with its own style and focus.

In such a situation, it is of significance to have a book of relatively wide scope to introduce the basic subjects in soliton theory, and to help students and practitioners who wish to reach the frontier of research. The present book is written with this purpose in mind.

This book is written by nine contributors, and an effort has been made to ensure as far as possible that the chapters are independent. From the contents, the reader can trace the main subjects from the original physical problems to the basic mathematical approaches, including analytic methods and numerical methods. It should be mentioned that each author has made a systematic study in the corresponding field, and so each contribution reflects this experience. Each author attempts to demonstrate first the basic concepts and main problems, then introduces more recent research and his or her own results. We hope that this book will succeed in accomplishing the above-mentioned goals.

Soliton theory has become extremely rich, and this book cannot cover all aspects. For example the Riemann-Hilbert method and the algebraic-geometric method are not discussed. Nor are the various concrete applications to technical problems.

This book has been written for specialists, teachers and students in mathematics and physics.

Gu Chaohao