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Functional Calculus of Pseudo-Differential Boundary Problems

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PREFACE

The theory of pseudo-differential operators has been developed through the last three decades as a powerful tool to handle partial differential equations. Here the pseudo-differential operators, and more generally the Fourier integral operators, include as special cases both the differential operators, their solution operators (integral operators), and compositions of these types. For equations on manifolds with boundary, Eskin, Vishik and Boutet de Monvel invented in particular the calculus of *pseudo-differential boundary operators*, that applies to elliptic boundary value problems.

The aim of the present book is to develop a *functional calculus* for such operators; i.e. to find the structure and properties of functions of these operators defined abstractly by functional analysis.

We consider in particular detail the exponential function of the operators, which leads to a treatment of parabolic evolution problems, and the complex powers of the operators, with applications to spectral theory; and we determine trace formulas and index formulas. The basic tool is a study of the *resolvent* of the operator, and this is worked out in the framework of a calculus of *pseudo-differential boundary problems depending on a parameter* $\mu \in \mathbb{R}_+$. The original parameter-independent theory is included as a special case, and our presentation may be used as an introduction to that theory. A further application of the theory is the treatment of singular perturbation problems; they contain a small parameter ε going to zero, corresponding to $\mu = \varepsilon^{-1}$ going to infinity.

The work was begun during a stay at the Ecole Polytechnique in 1979. At that time, we expected the resolvent analysis to take a few months (- with a sound knowledge of the Boutet de Monvel theory, it should be an easy matter to establish corresponding results in cases with a parameter -), but the task turned out to be not quite so simple. A first version of our calculus was written up in a series of reports from Copenhagen University in 1979-80 [Grubb 11], and much of that is used here (in shortened form). However, it also had some flaws: On one hand, the hypotheses needed to go beyond the most classical boundary conditions were incomplete, and on the other hand, we later found a way to eliminate a certain "loss of regularity $\frac{1}{2}$". Brief accounts of the theory have been given in [Grubb 12-15], where [Grubb 15] corrects earlier defects.

The present work contains much more, both in the form of explicit information on the structure of the boundary problems to which the theory applies, an amelioration of the calculus, and developments of consequences of the theory. It has taken a long time to complete (partly because of the author's other University duties), but we hope that this has led to a maturing of the contents and elimination of disturbing errors.

Various people have been helpful to us during the process. The start of the work benefited from conversations with Charles Goulaouic and Louis Boutet de Monvel in Paris. In 1981, Denise Huet in Nancy told us of the connection with singular perturbation theory, which led to a clarification of the hypotheses; and Bert-Wolfgang Schulze and Stefan Rempel in Berlin showed much interest in the work. When it was in a final stage, Lars Hörmander in Stockholm helped us greatly with criticism and suggestions for improvements. We are very thankful to these and other colleagues that have shown interest, and we likewise thank the editor and referees of the Birkhäuser Progress in Mathematics Series for their encouragement.

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Copenhagen in April 1986,

Gerd Grubb

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