Representations of Integers as Sums of Squares

Emil Grosswald

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Preface

During the academic year 1980–1981 I was teaching at the Technion—the Israeli Institute of Technology—in Haifa. The audience was small, but consisted of particularly gifted and eager listeners; unfortunately, their background varied widely. What could one offer such an audience, so as to do justice to all of them? I decided to discuss representations of natural integers as sums of squares, starting on the most elementary level, but with the intention of pushing ahead as far as possible in some of the different directions that offered themselves (quadratic forms, theory of genera, generalizations and modern developments, etc.), according to the interests of the audience.

A few weeks after the start of the academic year I received a letter from Professor Gian-Carlo Rota, with the suggestion that I submit a manuscript for the *Encyclopedia of Mathematical Sciences* under his editorship. I answered that I did not have a ready manuscript to offer, but that I could use my notes on representations of integers by sums of squares as the basis for one. Indeed, about that time I had already started thinking about the possibility of such a book and had, in fact, quite precise ideas about the kind of book I wanted it to be.

Specifically, I had read with much pleasure a book by K. Zeller on *Summability* (Ergebnisse der Mathematik und ihrer Grenzgebiete No. 15, Springer-Verlag). What impressed me mainly was the completeness of the bibliographic references. I was moved to emulate this model and write a book on representations by sums of squares that would quote a comfortably large number of known results, occasionally with condensed proofs only, but with bibliographic references as complete as possible.

Professor Rota encouraged me to write such a text, and I proceeded. When the manuscript was completed, however, it came as a real surprise to me that, except for the attempt to have a complete bibliography, there was no resemblance whatsoever between my text and its model by Zeller.

The original draft profited greatly from suggestions made by Professors George Andrews (The Pennsylvania State University), Marvin Knopp (Temple University), and Olga Taussky-Todd (California Institute of Technology), as well as by an anonymous referee. Also, Professor Martin Kneser (University of Göttingen) read the whole manuscript at least twice, with incredible care, pointing out a large number of errors of omission as well as of commission. To all of them I express my deepest gratitude. Particular thanks are due to all colleagues, who called my attention to bibliographic items which had eluded me. I also thank Professor Rota; his encouragement was an essential element in the decision to develop my notes into the present text.

At a certain moment the original publisher appeared to have lost interest in this venture. I am happy that Springer-Verlag was receptive to the suggestion that it take over. Perhaps it is appropriate that the publishers of *Limitierungsverfahren* ... and of "Representations of integers as sums of squares" should be the same. I express my gratitude to Springer-Verlag for its support and cooperation.

Finally, I remember fondly my audience at the Technion: their keen interest was an important stimulus in the preparation of the notes that grew into this manuscript.

My visit at the Technion had been made possible by a Lady Davis Fellowship, for which I also express my gratitude.

May the reader have as much fun from this volume as the author had in writing it!

Narberth, Pennsylvania
May 22, 1984

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