Progress in Mathematics
Vol. 25

Edited by
J. Coates and
S. Helgason

Springer Science+
Business Media, LLC

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# Exterior Differential Systems and the Calculus of Variations

1983

Springer Science+
Business Media, LLC

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Library of Congress Cataloging in Publication Data

Griffiths, Phillip.
Exterior differential systems and the calculus of variations.

(Progress in mathematics ; v. 25)
Includes index.
1. Calculus of variations. 2. Exterior differential systems. I. Title. II. Series: Progress in mathematics (Cambridge, Mass.) ; v. 25.
QA316.G84 1982 515'.64 82-17878

CIP-Kurztitelaufnahme der Deutschen Bibliothek

Griffiths, Phillip A.:
Exterior differential systems and the calculus of variations / Phillip A. Griffiths. - Boston ; Basel ; Stuttgart : Birkhauser, 1982.
(Progress in mathematics ; Vol. 25)

NE: GT

ISBN 978-0-8176-3103-1 ISBN 978-1-4615-8166-6 (eBook)
DOI 10.1007/978-1-4615-8166-6


Originally published by Birkhäuser Boston in 1983

To the memory of my mother

Jeanette Field Griffiths

TABLE OF CONTENTS

## LIST OF COMMONLY USED NOTATIONS

(Note: The references for the undefined terms used below may be found in the index.)

| | |
|---|---|
| $A^*(X)$ | Exterior algebra of smooth differential forms on a manifold $X$ |
| $\{\Sigma\}$ | Algebraic ideal in $A^*(X)$ generated by a set $\Sigma$ of forms on $X$ |
| $(I,\omega)$ | Exterior differential system with independence condition |
| $V(I,\omega)$ | Set of integral manifolds of $(I,\omega)$ |
| $T_N(V(I,\omega))$ | Tangent space to $V(I,\omega)$ at $N$ |
| $(I,\omega;\varphi)$ | Variational Problem (cf. Chapter I, Sec. a) |
| $\Phi: V(I,\omega) \to \mathbb{R}$ | Functional on $V(I,\omega)$ |
| $\delta\Phi: T_N(I,\omega) \to \mathbb{R}$ | Differential of $\Phi$ |
| $V(I,\omega;[A,B]))$ | Subset of $V(I,\omega)$ given by endpoint conditions |
| $T_N(V(I,\omega;[A,B]))$ | Tangent space to $V(I,\omega;[A,B])$ |
| $\equiv \bmod I$ | Congruence modulo an ideal $I \subset A^*(X)$ |
| $\overline{\overline{\equiv}}$ | Congruence modulo the image of $I \wedge I \to A^*(X)$ (cf. (II.b.4)) |
| $\mathbb{P}E$ | Projectivization of a vector space $E$ |
| $\theta_N$ | Restriction of $\theta \in A^*(X)$ to a submanifold $N \subset X$ |
| $d\theta = \Theta$ | Exterior derivative of a differential form; little $\theta$ is frequently denoted by capital $\Theta$ |
| $F(\cdot)$ | Frame manifold |
| $L_v\varphi$ | Lie derivative of a form $\varphi$ along a vector field $v$ |
| $Y$ | Momentum space associated to $(I,\omega;\varphi)$ |
| $Q$ | Reduced momentum space associated to $(I,\omega;\varphi)$ |