Math into LaTeX

Third Edition

George Grätzer

Math into LaTeX

Third Edition

BIRKHÄUSER • SPRINGER
BOSTON NEW YORK

George Grätzer
Department of Mathematics
University of Manitoba
Winnipeg, MB R3T 2N2
Canada

Library of Congress Cataloging-in-Publication Data
Grätzer, George A.
Math into LaTeX / George Grätzer.–3rd ed.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-0-8176-4131-3 e-ISBN-13: 978-1-4612-2134-0
DOI: 10.1007/978-1-4612-2134-0

1. LaTeX (Computer file) 2. AMS-LaTeX. 3. Mathematics printing—Computer programs. 4. Computerized typesetting. I. Title

Z253.4.L38 G745 2000
686.2'2544536—dc21 00-036088
CIP

ISBN-13: 978-0-8176-4131-3 Printed on acid-free paper.

9 8 7 6 5 4 3 SPIN 11318521

Birkhäuser is part of *Springer Science+Business Media*

www.birkhauser.com

Short contents

Contents

List of tables

List of figures

Quick Finder

Preface to the third edition

Why a new edition?

The Internet

Just a few years ago, the Internet consisted of little more than e-mail, USENET, and FTP sites. The state-of-the-art in information technology was Gopher, a text-based system using hierarchical menus to organize documents. Today the World Wide Web dominates the headlines in major magazines and newspapers. Many journals now have electronic editions, and new journals published solely on the Internet are beginning to appear. E-books and e-learning have started to establish themselves. The popularity and ease of use of the World Wide Web make it one of the best ways to share LaTeX articles, reports, and books with a wider audience.

Part VI discusses the interaction between LaTeX and the Internet:

- Chapter 13 examines the role of the Internet as the main source for information about using and customizing LaTeX.
- Chapter 14 explains how to publish your own LaTeX articles, reports, and books on the World Wide Web.

New focus

This edition focuses on the "standard LaTeX." The first edition of this book (published in 1993) described AMS-LaTeX, version 1.1, and the `amsart` document style. AMS-LaTeX, version 1.1, was a stand-alone product that was incompatible with the standard LaTeX of the time, LaTeX 2.09. The second edition (in 1996) reported on the new LaTeX (then called LaTeX 2_ε) and the new LaTeX-compatible

AMS packages that replaced $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX, version 1.1, but the book still had an $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX–centric view.

This third edition is about LaTeX. Where necessary, I recommend that you use packages to extend LaTeX's capabilities. For typesetting mathematics, I strongly recommend that you use the AMS packages.

AMS packages, version 2.0

The American Mathematical Society released version 2.0 of the AMS packages in 1999. This third edition covers the changes made in this release.

Books

The first and second editions of this book dealt primarily with the tasks involved in writing articles. In Part V, the third edition addresses the issues that arise when creating longer documents. In addition to chapters on BibTeX and *MakeIndex*, I have added a new chapter on writing books. Appendix F illustrates the importance of choosing a well-designed book document class.

Other changes

LaTeX LaTeX 2_ε has been remarkably stable since its release in 1996, becoming the standard LaTeX (see Section C.1.2). Changes have been minor except for advances in using LaTeX with non-English languages (see the new Appendix E) and the widespread use of the PostScript CM and AMS fonts (see the new Section D.1).

Reorganization and additions Due to the new emphasis on writing books, a number of sections and subsections have moved from Chapter 2 and Chapters 6–8 to the new Chapter 12.

I have carefully revised the content to cover the changes made in LaTeX and the AMS packages. I have added new material based on my own experiences (since 1996, I have typeset roughly 1500 pages—including two books—using LaTeX) and in response to e-mail queries from readers of previous editions (as an example, see the revised Section 5.6.2 on the `split` subsidiary math environment). Most of the text has been rewritten and there are many minor corrections.

In 1999, my introductory book, *First Steps in LaTeX*, [30] was published; it is based on Part I of the second edition of this book. Part I of this third edition takes into account the rewriting and editing that was done for *First Steps*.

Illustrations I believe that a visual illustration of a complicated construct substantially cuts the learning curve. So I have almost doubled the number of illustrations. See, for example, the illustrations of aligned formulas in Section 1.6.2;

the new Section 5.1, a visual guide to multiline math formulas; and the two-page spread of bibliographic styles in Chapter 10.

Web enhanced In the introduction, I explain how I plan to keep you, the reader, up-to-date on changes to come via the Web.

Two recurring questions

When I hear from readers, there are two questions that come up again and again:

1. I do not have much time to spend learning the technical aspects of writing articles. Do I really need a book as large as this one?
2. Can you help me to get started from scratch, covering everything from installing a working LaTeX system to the rudiments of text editing?

My answer to the first question is no. You do not need to read the entire book to get started. If you only read Part I (the short course), the few pages discussing the top matter of an AMS document (Section 8.1), and those parts of the book that cover the types of mathematical expressions your work uses, you will be able to write a basic article. *Math into LaTeX* is as large as it is because it addresses the use of LaTeX for a wide range of users. You can be very selective about what you choose to read at first, and come back later for more detail as needed.

The second question is addressed in a very small way by a section in the introduction, *Setting the stage*. There are dozens of different LaTeX implementations and hundreds of text editors. Your environment will be based on the kind of computer you have (or have access to), what you need your LaTeX system to do, how much work you are willing to do to maintain the system, and how much money you are willing to spend. Sections 13.2 and 13.3 will help you select a LaTeX system that meets your needs.

Because of the complex choices involved, no one book can possibly cover all of the possible combinations. I assume that you have a working and up-to-date LaTeX system, that you know how to use some text-editing application (even Word will do), and that you know the basics of working with your computer's operating system.

George Grätzer

Introduction

Is this book for you?

This book is for the mathematician, physicist, engineer, scientist, or technical typist who has to learn how to typeset articles containing mathematical formulas.

Part I provides a quick introduction to LaTeX, so that you will be ready to type your first article (such as the sample article on pages 44–45) in a very short time. That is followed, in Parts II–IV, by a detailed exposition that provides you with a solid foundation in LaTeX, so that typing mathematical documents will become second nature.

You can find specific topics in the short table of contents, in the detailed table of contents, in the Quick Finder, or in the index. While the index is LaTeX-oriented, the Quick Finder lists the main topics mainly using the terminology utilized by word processing applications. For example, to find out how to italicize text, look under "italics" in the Quick Finder, and under `\emph` in the index.

Setting the stage

Watch someone type a mathematical article in LaTeX. You will see that

- *A text editor* is used to create a LaTeX *source file.* A source file (we will call it `first.tex`) might look like the following:

```
\documentclass{article}
\begin{document}
The hypotenuse: $\sqrt{a^{2} + b^{2}}$.  I can type math!
\end{document}
```

  Note that the source file `first.tex` is different from a typical word-processor file: All characters are displayed at the same size and in the same font.

- *Typeset the source file and view the result on their monitor* (the two corners indicate material that is shown as typeset by LaTeX):

The hypotenuse: $\sqrt{a^2 + b^2}$. I can type math!

- *Continue the editing cycle.* You will go back and forth between the source file and the typeset version, making changes and observing the results of those changes.
- *Print the file.* Once you are satisfied with the typeset version, you can print the document, creating a paper version of the typeset article.

Unfortunately, I cannot tell you exactly how your particular text editor works, or how typesetting and printing is done on your system. Just as there are many text editors (ranging from the ancient `vi` to modern editors with graphical user interfaces), there are many LaTeX setups, each with its own unique installation and a different way of typesetting and printing. However, the following two examples should give you some idea of the process.

Example 1: UNIX

UNIX commands are typed at a *shell prompt* (such as `unix$`). The following command starts a text editor:

```
unix$ vi first.tex
```

Once the editor starts, you type the text of your article. When you are ready to typeset the article, save the file and quit the editor. Back at the shell prompt, typing

```
unix$ latex first
```

results in a series of messages scrolling up the screen as the file is typeset.

When this process is complete, you will have a DVI file, `first.dvi`, that may be viewed (in an X Window environment) by typing

```
unix$ xdvi first
```

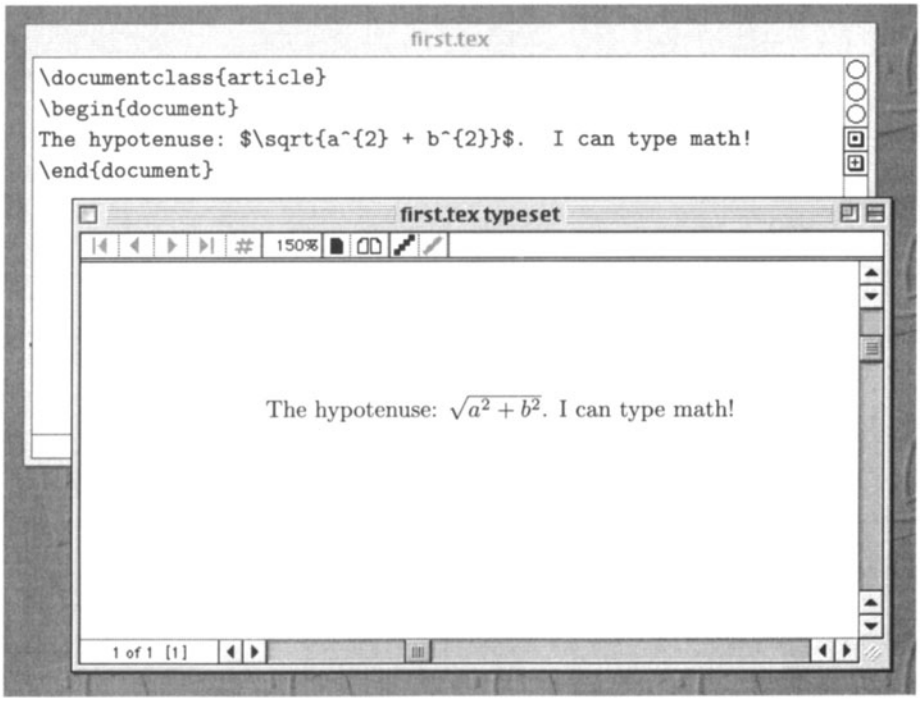
If changes must be made, you can return to the editor and make them, save and quit, then typeset and preview the file again. To print the DVI file, type a command such as the following at the shell prompt:

```
unix$ dvips first | lpr
```

Example 2: TEXTURES *on a Macintosh*

When you start the TEXTURES application on a Macintosh computer, a blank text-editing window appears. Type the text of your document in the window and save it as `first.tex`. When you are ready to typeset the document, make sure that the `LaTeX` format is selected in the `Typeset` menu, and then choose `Typeset` from the same menu.

A second window (titled `first.tex typeset`) appears, displaying the typeset version of your document:



To print your document, choose `Print...` from the `File` menu. To continue editing your document, simply click the mouse in the text-editing window to bring it to the front, and type. Depending on how you have set the options, the changes in the LaTeX source file may automatically appear in the window displaying the typeset version.

Choosing a LaTeX setup

In Sections 13.2 and 13.3, I briefly review a few of the most popular LaTeX setups. My best advice is to get the same LaTeX setup as that of a friend or colleague so they can help you get started. Many LaTeX implementations come on a CD-ROM or are downloadable from the Internet, and installation is often no more complicated

TEX

The TEX composition language was designed for typesetting mathematical and scientific articles and books, and can handle complex mathematical formulas as well as text: To get the formula $\int_0^\pi \sqrt{\alpha^2 + x^2}\,dx$, type

```
$\int_{0}^{\pi} \sqrt{\alpha^{2} + x^{2}}\,dx$
```

You do not have to worry about determining the size of the integral symbol or how to construct the square root symbol that covers $\alpha^2 + x^2$ because TEX does this for you!

A tremendous part of the appeal of the TEX language is that a source file is *plain text* (ASCII text), which is easy to transmit *electronically* to colleagues, coauthors, journals, editors, and publishers.

TEX is also *platform independent.* You may type the first version of a source file on a Macintosh computer; your coauthor may make improvements to the same file on a PC (a computer running Microsoft Windows); and the journal publishing the article may use a UNIX machine (a computer running a UNIX variant such as Solaris or Linux) to prepare the manuscript for printing.[2]

LATEX

LATEX was built on TEX's foundation, and has commands that are easier to use, a set of structural elements, and a larger set of diagnostic messages.

LATEX provides the following additional features:

- A document is divided into *logical units,* including an abstract, various sections and subsections, theorems, and a bibliography. The logical units are typed independently of one another. Once all the units have been typed, LATEX controls the *placement* and *formatting* of these elements.
- Line 4 of the sample article `intrart.tex` (on page 39) reads

  ```
  \documentclass{article}
  ```

  This line tells LATEX to load the *document class* `article`, which causes LATEX to format the document as a generic article. When submitting your article to a journal that is equipped to handle LATEX manuscripts (the number of such journals is increasing rapidly), the editor can simply substitute the name of the journal's document class to make the body of your article conform to the journal's design. (Depending on the journal, you may need to use the AMS article document class, `amsart`, in order to ensure that the front matter of your article also adapts.) Many journals make their own document classes available to prospective authors to use while preparing their articles.

[2] You may take this sentence as a definition of the three major computing platforms. We will only discuss tools that are available in some form on all three platforms.

- LaTeX's automatic numbering relieves you of tedious bookkeeping chores. Imagine that you have finished writing an article, with all of your theorems and equations numbered and properly cross-referenced. After a final reading, you must make some changes: Section 4 needs to be moved after Section 7, and some new theorems have to be inserted somewhere in the middle. Such minor changes used to be a major headache! But with LaTeX, it almost becomes a pleasure to make such changes: LaTeX automatically renumbers the sections, theorems, and equations in your article, and rebuilds the cross-references.
- With LaTeX you may use BibTeX, an application that helps you create and maintain bibliographic databases, so references do not have to be retyped for each article. BibTeX will select and format the needed references from your database.
- Compiling a large index is a big job. LaTeX users are assisted by *MakeIndex*, an application which makes this job easier.

The AMS packages

The AMS packages distill the American Mathematical Society's (AMS) years of experience in publishing mathematical journals and books; they add a host of features related to mathematical typesetting, especially the typesetting of multiline formulas and the production of finely tuned printed output. The AMS packages enhance LaTeX's capabilities in three different areas:

1. **Math.** The amsmath package adds a wide variety of tools for typesetting math, including

   - Powerful tools to deal with multiline math formulas. For instance, in the following formula, the equal signs (=) and the explanatory comments are vertically aligned:

$$\begin{aligned} x &= (x+y)(x+z) && \text{(by distributivity)} \\ &= x+yz && \text{(by Condition (M))} \\ &= yz. \end{aligned}$$

   - Numerous constructs for typesetting mathematical formulas, exemplified by the following:

$$f(x) = \begin{cases} -x^2, & \text{if } x < 0; \\ \alpha + x, & \text{if } 0 \le x \le 1; \\ x^2, & \text{otherwise.} \end{cases}$$

   - Special spacing rules for dozens of formula types; for example,

$$a \equiv b \pmod{\Theta}$$

typed inline becomes $a \equiv b \pmod{\Theta}$.

- Multiline subscripts, as in

$$\sum_{\substack{i^2+j^2=50 \\ i,\ j\le 10}} \sum \frac{x^i + y^j}{(i+j)!}$$

- User-defined symbols for typesetting math, such as

$$\operatorname{Trunc} f(x), \quad \hat{\hat{A}}, \quad {}^{*}\sum^{*}$$

- Formulas numbered in a variety of ways:
  - Automatically (with numbers)
  - Manually (with tags)
  - By groups, with a group number such as (2) and individual formulas numbered as (2a), (2b), and so on

2. **Document classes.** The AMS packages provide a number of document classes; the most important of which is the AMS article document class, `amsart`; it allows the input of title page information (e.g., author, address, e-mail address) as separate entities. As a result, a journal can typeset even the title page of an article according to its own specifications without having to retype any information.

   The AMS document classes provide the `proof` environment and three theorem styles: plain, definition, and remark. (See the `sampart.tex` sample article on pages 286–288: Theorem 1 uses the plain style, Definition 1 uses the definition style, and the Notation uses the remark style.)

   Many users also prefer the visual design of the `amsart` document class to the simpler design of the classic LaTeX `article` document class.

3. **Fonts.** The AMS packages provide hundreds of symbols for binary operations, binary relations, negated binary relations, arrows, extensible arrows, and so on (see the tables in Appendix A); there are also additional math alphabets such as blackboard bold, Euler Fraktur, Euler Script, and math bold and math bold italic. Here are just a few examples:

$$\leftleftarrows, \quad \blacktriangle, \quad \nexists, \quad \supsetneqq, \quad \mathbb{A}, \quad \mathfrak{p}, \quad \mathcal{E}$$

The AMS calls these enhancements $\mathcal{AMS}$-LaTeX (consisting of the math packages and the document classes) and AMSFonts (consisting of the font-related packages and the fonts themselves). In this book, to simplify the terminology, I refer to all these enhancements collectively as AMS *packages;* I use AMS *distribution* and AMS *enhancements* as synonyms.

I will point out in the text which commands are LaTeX commands and which are defined by AMS packages. References to AMS commands will also be indicated by the use of a symbol in the margin (such as the one shown here). A smaller version, Ⓐ, is used in the tables of Appendixes A and B and in the index.

What's in the book?

Just before this introduction is the **Quick Finder**, a brief index using mainly non-LaTeX terms.

Part I (**Chapter 1**) will help you get started quickly with LaTeX; if you read it carefully, you will be ready to type your own first article and to tackle LaTeX in more depth.

Part I guides you through

- Text markup, which is quite easy
- Math markup, which is not so straightforward; several sections ease you into mathematical typesetting, including
  - The basic building blocks of math formulas
  - How to build up a complex formula in simple steps
  - A formula gallery
  - Equations and multiline formulas
- The anatomy of an article
- How to set up an article template
- Typing your first article

Part II introduces the two most basic skills for writing with LaTeX in depth: *typing text* and *typing math.*

Chapters 2 and **3** introduce *text* and *displayed text.* Chapter 2 is especially important because when you type a LaTeX document, most of your time is spent typing text. The topics covered include special characters and accents, hyphenation, fonts, and spacing. Chapter 3 covers displayed text, including *lists* and *tables,* and for the mathematician, proclamations (theorem-like structures) and proofs.

Chapters 4 and **5** discuss *inline* and *displayed math.* Typing math is the heart of any mathematical typesetting system. Chapter 4 discusses this topic in detail, including basic constructs, operators, delimiters, building new symbols, fonts, and grouping equations. Chapter 5 presents one of the major contributions of the AMS packages: aligned multiline formulas. This chapter also discusses other forms of multiline formulas.

Part III discusses the parts of a LaTeX document. In **Chapter 6**, you learn about the structure of a LaTeX document. The most important topics are sectioning and cross-referencing. In **Chapter 7**, the most commonly used standard LaTeX document classes are presented: `article`, `report`, and `letter` (the `book` class is discussed in Chapter 12), along with a description of the standard LaTeX distribution.

In **Chapter 8**, we discuss the AMS document classes. In particular, I present the title page information for the AMS article document class and provide a description of the standard AMS distribution.

Chapter 8 also features the AMS sample article, `sampart.tex`, first in typeset form (pages 286–288), then in mixed form, juxtaposing the source file and the typeset article (pages 290–297). You can learn a lot about LaTeX and the AMS packages just by reading the source file one paragraph at a time and seeing how that paragraph is typeset by LaTeX.

Part IV (**Chapter 9**) introduces techniques to *customize* LaTeX to speed up the typing of source files and the typesetting of documents: user-defined commands, user-defined environments, and custom formats. You will learn how parameters that affect LaTeX's behavior are stored in *counters* and *length commands*, how to change them, and how to design your own custom lists.

Chapter 9 also contains a version of the AMS sample article utilizing the user-defined commands collected in `lattice.sty`.

In **Part V** (**Chapters 10** and **11**), we will discuss longer documents, which have special needs. Two applications, contained in the standard LaTeX distribution, BibTeX and *MakeIndex* make compiling large bibliographies and indexes much easier.

I present the LaTeX and the AMS book document classes in **Chapter 12** along with the *dos and don'ts* of book writing in LaTeX.

Part VI deals with LaTeX and the Internet. **Chapter 13** discusses where to find useful LaTeX-related information on the Internet. The main topics are:

- Obtaining files from the Internet
- CTAN, the Comprehensive TeX Archive Network
- Obtaining the LaTeX distribution and the AMS packages
- Getting the sample files for this book
- Some commercial TeX implementations
- Freeware and shareware TeX implementations
- TeX user groups and the AMS
- Important LaTeX-related FTP and Web sites

You can share your LaTeX articles, reports, and books by putting them on the Web so that others can view, read, download, and print them. **Chapter 14** tells you how.

You will probably find yourself referring to **Appendices A** and **B** time and again: They contain the math and text symbol tables.

Appendix C relates some historical background material on LaTeX: how it developed and how it works. **Appendix D** is a brief introduction to the use of PostScript fonts in a LaTeX document.

Appendix E briefly describes the use of LaTeX for languages other than American English. **Appendix F** shows a few pages from a book typeset with a Springer-Verlag book document class along with excerpts from the source document.

Appendix G will help orient those people who have previously worked with

(Plain) TEX, LATEX, version 2.09, AMS-TEX, or AMS-LATEX, version 1.x. Some tips are given to smooth your transition to using the current standard LATEX and the AMS packages. Finally, **Appendix H** points you towards some areas for further study.

Mission statement

This book is a guide for typesetting mathematical documents within the constraints imposed by LATEX, an elaborate system with hundreds of rules. LATEX allows you to perform almost any mathematical typesetting task through the appropriate application of its rules. You can customize LATEX (as it was designed to be modified) by introducing user-defined commands and environments and by changing LATEX parameters.

You can also extend LATEX by invoking packages that accomplish special tasks: One such set of packages from the AMS plays an important role in this book—as it should in any book dealing with mathematical typesetting.

It is not my goal to teach you

- How to modify LATEX code to change LATEX's behavior
- How to write TEX code to create your own packages (LATEX extensions)
- How to design beautiful documents (writing document classes)

The definitive book on the first topic is Michel Goossens, Frank Mittelbach, and Alexander Samarin's *The LATEX Companion* [17]. The second and third topics still await authoritative books.

A recommendation

I strongly recommend that you use the `amsart` document class for all your articles. Begin each article with the lines

```
\documentclass{amsart}
\usepackage{amssymb,latexsym}
\begin{document}
```

and you can ignore all of the discussions in this book about LATEX commands versus AMS commands, and LATEX fonts and the latexsym package versus AMS fonts and the amssymb package.

Some of you may not be able to follow this recommendation, including those who work with older installations whose system managers cannot or will not install a newer version of LATEX or the AMS packages, and those who are forced to use a publisher's document class file that is not compatible with the AMS packages. But most users of LATEX who typeset documents with significant amounts of math will find that using the `amsart` document class and loading amssymb and latexsym make their work easier.

Keeping up-to-date

Like most computer-related subjects, the material in this book is subject to change over time. While LaTeX itself may not change much until the advent of LaTeX3, there is a new version of the amsmath package on the horizon, introducing a variant of the `equation` environment that will automatically break long formulas into shorter lines. Chapter 13 deals with the Internet, which is in a state of constant flux. To keep you up-to-date, I am maintaining a Web page to track these changes for you. To find this page, go to my home page,
`http://www.maths.umanitoba.ca/homepages/gratzer/`
and follow the links `LaTeX books` and `MiL Update`. Or go directly to
`http://www.maths.umanitoba.ca/homepages/gratzer/LaTeXBooks/milupdate.html`

Conventions

To make this book easy to read, I use some simple conventions:

- Explanatory text is set in this typeface: Galliard.
- `Computer Modern typewriter is used to show what you should type (as well as messages from LaTeX). All the characters in this typeface have the same width, making it easy to recognize.`
- I also use Computer Modern typewriter to indicate
  - Commands (`\parbox`)
  - Environments (`align`)
  - Documents (`intrart.tex`)
  - Document classes (`article`)
  - Document class options (`draft`)
  - Directories or folders (`work`)
- The names of *packages,* which are extensions of LaTeX, are set in a sans-serif typeface (amsmath).
- When I show you how something looks when typeset, I use Computer Modern, TeX's standard typeface:

I think you will find this typeface sufficiently different from the other typefaces I have used (the strokes are much lighter) so that you should not have much difficulty recognizing typeset LaTeX material. When the typeset material is a separate paragraph (or paragraphs), corner brackets in the margin set it off from the rest of the text—unless it is a single displayed formula.

- For explanations in the text, such as

  Compare iff with iff, typed as `iff` and `if{f}`, respectively.

the same typefaces are used. Because they are not set off spatially, it may be a little more difficult to see that iff is set in Computer Modern roman, whereas `iff` is set in the Computer Modern typewriter typeface.

- I usually introduce commands with examples, such as

  ```
  \\[22pt]
  ```

  However, it is sometimes necessary to define the syntax of a command more formally. For instance,

  ```
  \\[length]
  ```

  where *length* is a *placeholder* representing the value you have to supply. I use the Computer Modern typewriter italic font for placeholders.
- I use the term *directory* to mean both directory and folder.

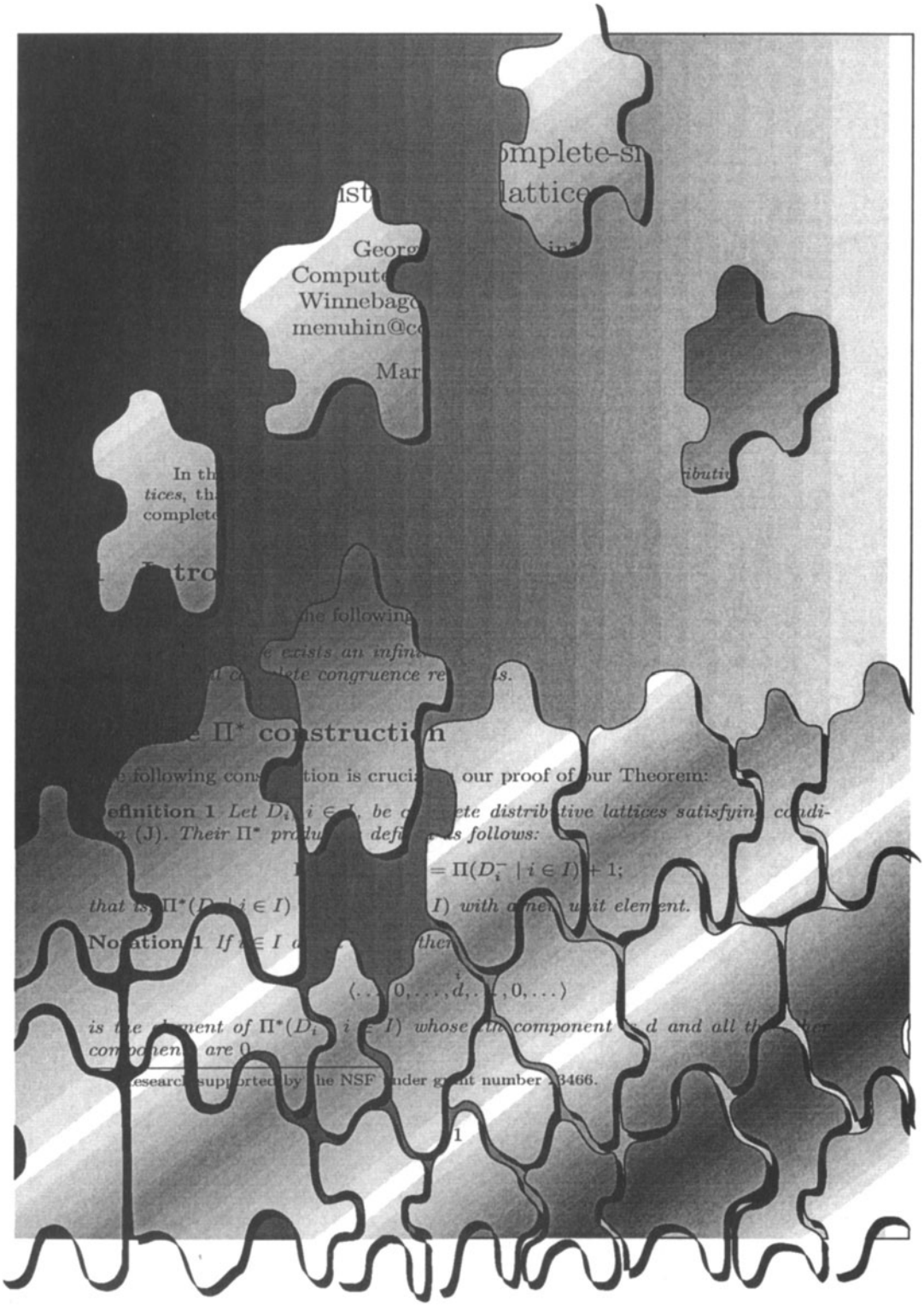
Acknowledgments

This book is based, of course, on its previous editions. I would like to thank the many people, too numerous to list here again, who read and reread those earlier manuscripts.

I received professional reports on the manuscript from Barbara Beeton, Nandor Sieben, and Ferenc Wettl. Arthur Ogawa commented on Part I. The chapter on BibTeX has been carefully reviewed—again—by Oren Patashnik (the author of BibTeX); the chapter on the Web was read by Sebastian Rahtz (the author of the hyperref package and coauthor of the *The LaTeX Web Companion* [19]); the chapter on books was read by Fred Bartlett (Electronic Publishing, Springer-Verlag New York).

Claire M. Connelly did an outstanding job editing the manuscript, far and beyond the call of duty; in addition to editing the text and making suggestions for improvements, she redesigned the tables and updated the index. Melissa O'Neill provided two ingenious Perl scripts for cleaning up the index. Ann Kostant demonstrated that publishers care; this complex project greatly benefited from her guidance and editorial advice. Elizabeth Loew carefully guided the manuscript to publication.

George Grätzer
E-mail: gratzer@cc.umanitoba.ca
Home page: http://www.maths.umanitoba.ca/homepages/gratzer/



mplete-s
ist lattice
Georg in*
Compute
Winnebag
menuhin@c
Mar
In th ibuti
tices, th
complet
Intro
he followin
exists an infin
t complete congruence re s.
e Π^* construction
following cons tion is crucia our proof of our Theorem:
efinition 1 *Let D_i, $i \in I$, be c ete distributive lattices satisfying condi-*
n (J). *Their Π^* produ defi s as follows:*
$$= \Pi(D_i^- \mid i \in I) + 1;$$
that is, $\Pi^*(D_i \mid i \in I)$ I) with a ne unit element.
Notation 1 *If $i \in I$ ther*
$$\langle \ldots, 0, \ldots, \overset{i}{d}, \ldots, 0, \ldots \rangle$$
is the element of $\Pi^*(D_i \mid i \in I)$ whose ith component is d and all the other components are 0.
esearch supported by the NSF under grant number 23466.