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Random Sets

Theory and Applications

With 39 Illustrations

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FOREWORD

This IMA Volume in Mathematics and its Applications

RANDOM SETS: THEORY AND APPLICATIONS

is based on the proceedings of a very successful 1996 three-day Summer Program on "Application and Theory of Random Sets." We would like to thank the scientific organizers: John Goutsias (Johns Hopkins University), Ronald P.S. Mahler (Lockheed Martin), and Hung T. Nguyen (New Mexico State University) for their excellent work as organizers of the meeting and for editing the proceedings. We also take this opportunity to thank the Army Research Office (ARO), the Office of Naval Research (ONR), and the Eagan, Minnesota Engineering Center of Lockheed Martin Tactical Defense Systems, whose financial support made the summer program possible.

Avner Friedman

Robert Gulliver

PREFACE

"Later generations will regard set theory as a disease from which one has recovered."

– Henri Poincaré

Random set theory was independently conceived by D.G. Kendall and G. Matheron in connection with stochastic geometry. It was however G. Choquet with his work on capacities and later G. Matheron with his influential book on *Random Sets and Integral Geometry* (John Wiley, 1975), who laid down the theoretical foundations of what is now known as the theory of *random closed sets.* This theory is based on studying probability measures on the space of *closed subsets* of a locally compact, Hausdorff, and separable base space, endowed with a special topology, known as the *hit-or-miss topology.* Random closed sets are just random elements on these spaces of closed subsets. The mathematical foundation of random closed sets is essentially based on *Choquet's capacity theorem*, which characterizes distribution of these set-valued random elements as nonadditive set functions or "nonadditive measures." In theoretical statistics and stochastic geometry such nonadditive measures are known as *infinitely monotone*, *alternating capacities of infinite order*, or *Choquet capacities*, whereas in expert systems theory they are more commonly known as *belief measures*, *plausibility measures*, *possibility measures*, etc. The study of random sets is, consequently, inseparable from the study of nonadditive measures.

Random set theory, to the extent that is familiar to the broader technical community at all, is often regarded as an obscure and rather exotic branch of pure mathematics. In recent years, however, various aspects of the theory have emerged as promising new theoretical paradigms for several areas of academic, industrial, and defense-related R&D. These areas include stochastic geometry, stereology, and image processing and analysis; expert systems theory; an emerging military technology known as "information fusion;" and theoretical statistics.

Random set theory provides a solid theoretical foundation for certain image processing and analysis problems. As a simple example, Fig. 1 illustrates an image of an object (a cube), corrupted by various noise processes, such as clutter and occlusions. Images, as well as noise processes, can be modeled as random sets. Nonlinear algorithms, known collectively as *morphological operators*, may be used here in order to provide a means of recovering the object from noise and clutter. Random set theory, in conjunction with *mathematical morphology*, provides a rigorous statistical foundation for nonlinear image processing and analysis problems that is analogous to that of conventional linear statistical signal processing. For example, it allows one to demonstrate that there exist optimal algorithms that recover images from certain types of noise processes.

In *expert systems theory*, random sets provide a means of modeling and

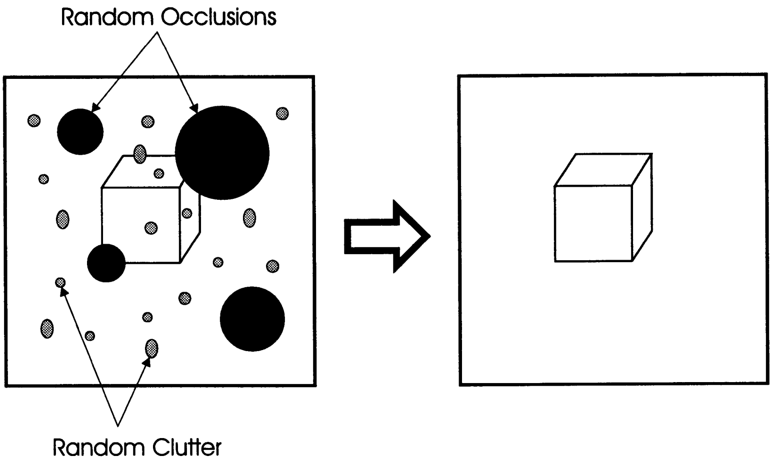

FIG. 1. *Random sets and image processing.*

manipulating evidence that is imprecise (e.g., poorly characterized sensor signatures), vague or fuzzy (e.g., natural language statements), or contingent (e.g., rules). In Fig. 2, for example, we see an illustration of a natural-language statement such as "*Gustav is NEAR the tower.*" Each of the four (closed) ellipses represents a plausible interpretation of the concept "*NEAR the tower*," and the numbers p_1, p_2, p_3, p_4 represent the respective beliefs that these interpretations of the concept are valid. A discrete random variable that takes the four ellipses as its values, and which has respective probabilities p_1, p_2, p_3, p_4 of attaining those values, is a random set representative of the concept.

Random sets provide also a convenient mathematical foundation for a statistical theory that supports *multisensor, multitarget information fusion.* In Fig. 3, for example, an unknown number of unknown targets are being interrogated by several sensors whose respective observations can be of very diverse type, ranging from statistical measurements generated by radars to English-language statements supplied by human observers. If the sensor suite is interpreted as a single sensor, if the target set is interpreted as a single target, and if the observations are interpreted as a single *finite-set* observation, then it turns out that problems of this kind can be attacked using direct generalizations of standard statistical techniques by means of the theory of random sets.

Finally, random set theory is playing an increasingly important role in *theoretical statistics.* For example, suppose that a continuous but random voltage is being measured using a digital voltmeter and that, on the basis of the measured data, we wish to derive bounds on the expected value of the original random variable, see Fig. 4. The observed quantity is a random subset (specifically, a random interval) and the bounds can be expressed in terms of certain nonlinear integrals, called *Choquet integrals*, computed

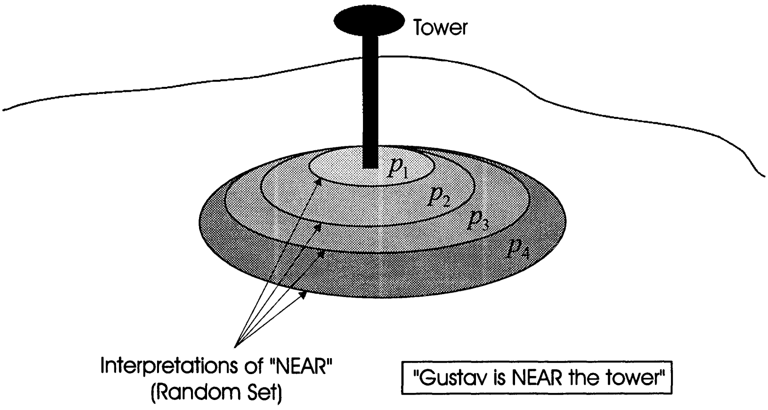

FIG. 2. *Random sets and expert systems.*

with respect to nonadditive measures associated with that random subset.

On August 22–24, 1996, an international group of researchers convened under the auspices of the *Institute for Mathematics and Its Applications* (IMA), in Minneapolis, Minnesota, for a scientific workshop on the "Applications and Theory of Random Sets." To the best of our knowledge this was the first scientific gathering in the United States, devoted primarily to the subject of random sets and allied concepts. The immediate purpose of the workshop was to bring together researchers and other parties from academia, industry, and the U.S. Government who were interested in the potential application of random set theory to practical problems of both industrial and government interest. The long-term purpose of the workshop was expected to be the enhancement of imaging, information fusion, and expert system technologies and the more efficient dissemination of these technologies to industry, the U.S. Government, and academia.

To accomplish these two purposes we tried to bring together, and encourage creative interdisciplinary cross-fertilization between, three communities of random-set researchers which seem to have been largely unaware of each other: theoretical statisticians, those involved in imaging applications, and those involved in information fusion and expert system applications. Rather than "rounding up the usual suspects"–a common, if incestuous, practice in organizing scientific workshops–we attempted to mix experienced researchers and practitioners having complementary interests but who, up until that time, did not have the opportunity for scientific interchange.

The result was, at least for a scientific workshop, an unusually diverse group of researchers: theoretical statisticians; academics involved in applied research; personnel from government organizations and laboratories, such as the National Institutes of Health, Naval Research and Development, U.S. Army Research Office, and USAF Wright Labs, as well as industrial R&D engineers from large and small companies, such as Applied Biomath-

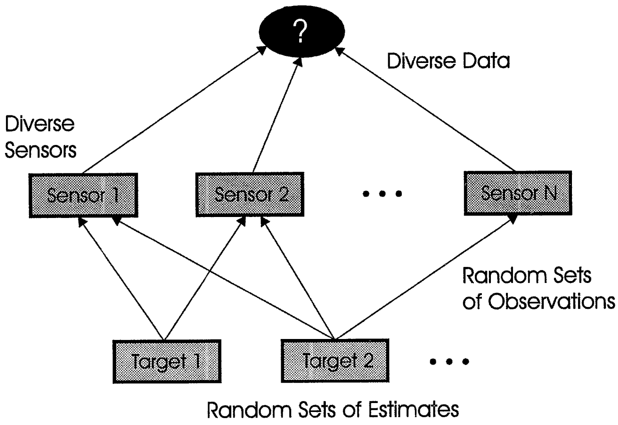

FIG. 3. *Random sets and information fusion.*

ematics, Data Fusion Corporation, Lockheed Martin, Neptune and Company, Oasis Research Center, Raytheon, Texas Instruments, and Xerox. The papers in this volume reflect this diversity. A few papers are tutorial in nature, some are detailed mathematical treatises, some are summary overviews of an entire subject, and still others are investigations rooted in practical engineering intuition.

The workshop was structured into three sessions, devoted respectively to the following topic areas, each organized and chaired by one of the editors:

- *Image Modeling and Analysis* (J. Goutsias).
- *Information/Data Fusion and Expert Systems* (R.P.S. Mahler).
- *Theoretical Statistics and Expert Systems* (H.T. Nguyen).

Each session was preceded by a plenary presentation given by a researcher of world standing:

- Ilya Molchanov, University of Glasgow, Scotland.
- Jean-Yves Jaffray, University of Paris VI, France.
- Ulrich Höhle, Bergische Universität, Germany.

The following institutions kindly extended their support to this workshop:

- *U.S. Office of Naval Research*, Mathematical, Computer, and Information Sciences Division.
- *U.S. Army Research Office*, Electronics Division.
- *Lockheed Martin*, Eagan, Minnesota Engineering Center.

The editors wish to express their appreciation for the generosity of these sponsors. They also extend their special gratitude to the following individuals for their help in ensuring success of the workshop: Avner Friedman,

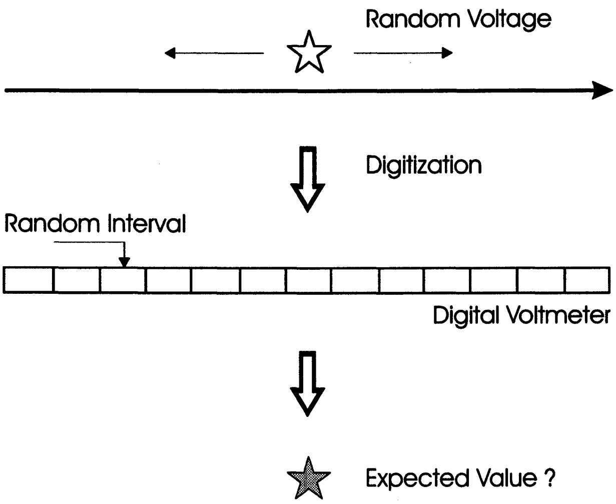

FIG. 4. *Random sets and theoretical statistics.*

IMA, Director; Julia Abrahams, Office of Naval Research; William Sander, Army Research Office; Wesley Snyder, North Carolina State University; Marjorie Hahn, Tufts University; Larry Wasserman, Carnegie-Mellon University; Charles Mills, Lockheed Martin, Director of Engineering; Amy Cavanaugh, IMA, Workshop Coordinator; and John Schepers, IMA, Workshop Financial Coordinator.

In committing these proceedings to the attention of the larger scientific and engineering community, the editors hope that the workshop will have thereby contributed to one of the primary goals of IMA: facilitating creative interchange between statisticians, scientists, and academic and industrial engineers in technical domains of potential practical significance.

John Goutsias
Ronald P.S. Mahler
Hung T. Nguyen

Workshop on
Applications and Theory of Random Sets
Institute for Mathematics and its Applications (IMA)
University of Minnesota
Minneapolis, Minnesota
August 22–24, 1996

Participants
(From left-to-right)

Lower Row: Scott Ferson, Wesley Snyder, Yidong Chen, Bert Fristedt, John Handley, Sinan Batman, Edward Dougherty, Nikolaos Sidiropoulos, Dan Schonfeld, I.R. Goodman, Wolfgang Kober, Stan Music, Ronald Mahler, Jean-Yves Jaffray, Elbert Walker, Carol Walker, Hung Nguyen, John Goutsias

Upper Row: Robert Launer, Paul Black, Tonghui Wang, Shozo Mori, Robert Taylor, Ulrich Hőehle, Ilya Molchanov, Michael Stein, Krishnamoorthy Sivakumar, Fred Daum, Teddy Seidenfeld

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