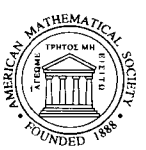
MATHEMATICAL
Surveys and Monographs

Volume 40, Number 4

The Classification of the Finite Simple Groups, Number 4

Part II, Chapters 1–4: Uniqueness Theorems

Daniel Gorenstein
Richard Lyons
Ronald Solomon



American Mathematical Society
Providence, Rhode Island

Contents

Preface xiii

PART II, CHAPTERS 1–4: UNIQUENESS THEOREMS

Chapter 1. General Lemmas 1
1. Uniqueness Elements 1
2. Permutation Groups 3
3. Two Theorems on Doubly Transitive Groups 7
4. Miscellaneous Lemmas 12

Chapter 2. Strongly Embedded Subgroups and Related Conditions on Involutions 19
1. Introduction and Statement of Results 19
2. The Subsidiary Theorems 25
3. The Basic Setup and Counting Arguments 27
4. Reduction to the Simple Case: Theorem 1 33
5. p-Subgroups Fixing Two or More Points: Theorem 2 38
6. A Reduction: Corollary 3 42
7. Double Transitivity and $X_{\alpha\beta\beta^z}$: Theorem 4 44
8. Reductions for Theorem SE 50
9. Good Subgroups of V Exist 54
10. The Structures of V and D 58
11. Proof of Theorem 6 (and Theorem SE) 62
12. The Group $L = \langle z, t, [O_{2'}(D), t] \rangle$ 65
13. The Unitary Case 72
14. The Suzuki Case 74
15. The Linear Case 79
16. The Structure of D and $C_X(u)$ 82
17. The Proof of Theorem ZD 88
18. The Weak 2-Generated 2-Core 91
19. The J_2 Case 95
20. Bender Groups as 2-Components 98
21. Theorem SU: The 2-Central Case 101
22. The J_1 Case 103
23. The $2A_9$ Case 105
24. Theorem SA: Strongly Closed Abelian 2-Subgroups 109
25. Theorem SF: Terminal Bender Components 112
26. Theorem SF: Product Disconnection 119
27. Theorem A_9 127

Chapter 3. p-Component Uniqueness Theorems 131
1. Introduction 131
2. Theorem PU_1: The Subsidiary Results 137
3. Exceptional Triples and $\mathcal{K}$-Group Lemmas 138
4. M-Exceptional Subgroups of G 142
5. Centralizers of Elements of $\mathcal{I}_p(K^*)$ 147
6. The Proofs of Theorems 1 and 2 150
7. Theorem 3: The Nonsimple Case 153
8. Theorem 3: The Simple Case 163
9. Theorem 4: Preliminaries 165
10. Normalizers of p-Subgroups of K 168
11. The Case $\overline{K} \notin \mathcal{SE}(p)$ 171
12. The $\mathcal{SE}(p)$ case 172
13. Theorem 5: The M-Exceptional Case 179
14. The Residual M-Exceptional Cases 188
15. Completion of the Proof of Theorem PU_1: The p-Rank 1 Case 193
16. Reductions to Theorem PU_1 194
17. Further Reductions: The Non-Normal Case 201
18. Theorem PU_2: The Setup 206
19. The Case $r < p$: A Reduction 208
20. The Case $r = 1$: Conclusion 211
21. The Case $r = p = 2$: A Reduction 214
22. The Case $r = p = 2$, $\overline{K} \cong L_2(q)$ 216
23. Theorem PU_3: The Simple Case 220
24. The Residual Simple Cases 223
25. The Nonsimple Case 224
26. Theorem PU_4 226
27. Corollaries PU_2 and PU_4 227
28. Aschbacher's Reciprocity Theorem 228
29. Theorem PU_5 232

Chapter 4. Properties of $\mathcal{K}$-Groups 237
1. Automorphisms 237
2. Schur Multipliers and Covering Groups 238
3. Bender Groups 245
4. Groups of Low 2-Rank 253
5. Groups of Low p-Rank, p Odd 256
6. Centralizers of Elements of Prime Order 257
7. Sylow 2-Subgroups of Specified $\mathcal{K}$-Groups 268
8. Disconnected Groups 271
9. Strongly Closed Abelian Subgroups 276
10. Generation 283
11. Generation and Terminal Components 293
12. Preuniqueness Subgroups and Generation 302
13. 2-Constrained Groups 328
14. Miscellaneous Results 331

Background References 333

Expository References 334

Errata for Number 3, Chapter I_A: Almost Simple $\mathcal{K}$-Groups 335

Glossary 338

Index of Terminology 340