

Undergraduate Texts in Mathematics

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Undergraduate Texts in Mathematics

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(continued after index)

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Discrete Probability



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*This book is dedicated
to my parents.*

Preface

Students of literature may have an advantage in understanding what this book is about. Noticing the many references to games and gambling, these students will immediately ask the right question: For what are the games and gambling a metaphor? The answer is the many events in the real world, some of them of great importance, in which chance plays a role. In almost all areas of human endeavor, a knowledge of the laws of probability is essential to success. It is not possible to work with actual uses of probability theory here. In the first place, applications in one area will be of limited interest to students of other areas. Second, in order to consider an application of probability in some field of study, one must know something about that field. For example, to understand how random walks are used by chemists, one must know the necessary chemistry. Since we do want to explain probability theory primarily by presenting examples, we choose to discuss simple things, like throwing a pair of dice. The examples stand in place of the many practical situations that are too complicated to discuss in this book.

Certainly the book is not intended as a guide to gambling, or even as a warning against gambling. However, before dropping the subject of gambling, a certain comment may be in order. We may as well quote the words used by DeMoivre, in his book, *The Doctrine of*

Chances, in similar circumstances. (Information about the persons quoted in this preface may be found in the body of the book.) Note that the doctrine of chances refers to what we now call probability theory. DeMoivre wrote, "There are many People in the World who are prepossessed with an Opinion, that the Doctrine of Chances has a Tendency to promote Play, but they soon will be undeceived, if they think fit to look into the general Design of this Book."

While applications abound, the present work can be read as a study in pure mathematics. The style is very formal, but we do have definitions, axioms and theorems; proofs of the theorems are given for those who care, or are obliged by an instructor, to read them. To quote DeMoivre again, "Another use to be made of this Doctrine of Chances is, that it may serve in Conjunction with the other parts of Mathematicks, as a fit Introduction to the Art of Reasoning; it being known by experience that nothing can contribute more to the attaining of that Art, than the consideration of long Train of Consequences, rightly deduced from undoubted Principles, of which this Book affords many Examples. To this may be added, that some of the Problems about Chance having a great Appearance of Simplicity, the Mind is easily drawn into a belief, that their Solution may be attained by meer Strength of natural good Sense; which generally proving otherwise, and the Mistakes occasioned thereby being not unfrequent, 'tis presumed that a Book of this Kind, which teaches to distinguish Truth from what seems so nearly to resemble it, will be look'd upon as a help to good Reasoning."

As we just noted, this book is about a part of mathematics—applicable mathematics to be sure, but the applications are not discussed here. Putting applications to one side, we look at the other side. There we find philosophy, which we also cannot discuss here: Is there such a thing as chance in the real world, or is everything predestined? Let us take a moment to at least indicate which topics we are not going to discuss.

One point of view was expressed by Montmort, writing in French in the eighteenth century. In translation what he says is, "Strictly speaking, nothing depends on chance; when we study nature, we are completely convinced that its Author works in a general and uniform way, displaying infinite wisdom and foreknowledge. Thus to attach a philosophically valid meaning to the word "chance", we

must believe that everything follows definite rules, which for the most part are not known to us; thus to say something depends on chance is to say its actual cause is hidden. According to this definition we may say that the life of a human being is a game where chance rules."

On the other hand, at the present time, there are those who claim that in quantum mechanics we find situations in which there is nothing but blind chance, with no causality behind it; all that actually exists before we make an observation is certain probabilities. We need not go into this point of view here, beyond noting the following: If we see from experiments that the rules of probability theory are followed in certain situations, for many purposes it is not necessary to know why they are followed. Be that as it may, the examples which appear in this book are all of the kind Montmort had in mind. We speak of whether a coin falls heads as a random event, because we do not have the detailed information necessary to compute from the laws of physics how the coin will fall.

* * *

How much of the book should be covered in a one semester course, or a two quarter course, is obviously up to the instructor. Likewise the instructor must decide how much to emphasize theorems and proofs.

Certain exercises are marked with two asterisks to indicate unusual difficulty. But who can say just how hard a problem is? What is certain is that the exercises as a whole were deliberately selected to cover a range of difficulties.

The book developed gradually over a long period of time. During that time the author was greatly aided by discussion of the work with his colleagues, many of whom suffered the misfortune of teaching using inadequate preliminary versions of the book; many flaws can to light this way. The author is most grateful for the help of these colleagues in the Department of Mathematics & Statistics of the State University of New York at Albany, especially George Martin, and for the help of the staff of that department.

Hugh Gordon

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