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Integral Equations**

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Preface

This monograph is the second volume of a graduate text book on the modern theory of linear one-dimensional singular integral equations. Both volumes may be regarded as unique graduate text books.

Singular integral equations attract more and more attention since this class of equations appears in numerous applications, and also because they form one of the few classes of equations which can be solved explicitly.

The present book is to a great extent based upon material contained in the second part of the authors' monograph [6] which appeared in 1973 in Russian, and in 1979 in German translation. The present text includes a large number of additions and complementary material, essentially changing the character, structure and contents of the book, and making it accessible to a wider audience.

Our main subject in the first volume was the case of closed curves and continuous coefficients. Here, in the second volume, we turn to general curves and discontinuous coefficients.

We are deeply grateful to the editor Professor G. Heinig, to the translator Dr. S. Roch, and to the typist Mr. G. Lillack, for their patient work.

The authors

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