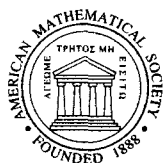


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**Ordinary Differential
Equations with
Constant Coefficient**

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