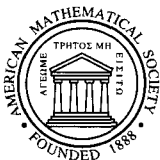


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**Modern Aspects
of Linear Algebra**

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