

Global Optimization in Action

*Continuous and Lipschitz Optimization:
Algorithms, Implementations and Applications*

by

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Kluwer Academic Publishers

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-1-4419-4751-2
DOI 10.1007/978-1-4757-2502-5

Printed on acid-free paper

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Softcover reprint of the hardcover 1st edition 1996

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