Orthogonal Polynomials Computation and Approximation

Walter Gautschi Purdue University



OXFORD UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide in

Oxford New York

Auckland Bangkok Buenos Aires Cape Town Chennai Dares Salaam Delhi Hong Kong Istanbul Karachi Kolkata Kuala Lumpur Madrid Melbourne Mexico City Mumbai Nairobi São Paulo Shanghai Taipei Tokyo Toronto

Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries

> Published in the United States by Oxford University Press Inc., New York

> > © Oxford University Press 2004

The moral rights of the author have been asserted

Database right Oxford University Press (maker)

First published 2004

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, or under terms agreed with the appropriate reprographics rights organization. Enquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above

You must not circulate this book in any other binding or cover and you must impose this same condition on any acquirer

A catalogue record for this title is available from the British Library

Library of Congress Cataloging in Publication Data

(Data available) ISBN 0 19 850672 4

10 9 8 7 6 5 4 3 2 1

Typeset by the author using LAT_FX

Printed in Great Britain on acid-free paper by Biddles Ltd. www.biddles.co.uk

CONTENTS

Pro	viii			
1	Basi	1		
	1.1	Ortho	1	
		1.1.1	Definition and existence	1
		1.1.2	Examples	4
	1.2	Prope	erties of orthogonal polynomials	6
		1.2.1	Symmetry	6
		1.2.2	Zeros	7
		1.2.3	Discrete orthogonality	8
		1.2.4	Extremal properties	8
	1.3	Three-term recurrence relation		10
		1.3.1	Monic orthogonal polynomials	10
		1.3.2	Orthonormal polynomials	12
		1.3.3	Christoffel–Darboux formulae	14
		1.3.4	Continued fractions	15
		1.3.5	The recurrence relation outside the support	
			interval	17
	1.4	Quadrature rules		20
		1.4.1	Interpolatory quadrature rules and beyond	21
		1.4.2	Gauss-type quadrature rules	22
	1.5	Classical orthogonal polynomials		26
		1.5.1	Classical orthogonal polynomials of a	
			continuous variable	27
		1.5.2	Classical orthogonal polynomials of a	
			discrete variable	32
	1.6	Kernel polynomials		35
		1.6.1	Existence and elementary properties	36
		1.6.2	Recurrence relation	38
	1.7	Sobol	ev orthogonal polynomials	40
		1.7.1	Definition and properties	41
		1.7.2	Recurrence relation and zeros	41
	1.8	Ortho	ogonal polynomials on the semicircle	43
		1.8.1	Definition, existence, and representation	43
		1.8.2	Recurrence relation	45
		1.8.3	Zeros	47
	1.9	Notes	to Chapter 1	49

2	Computational Methods				
	2.1 Moment-based methods				
		2.1.1	Classical approach via moment determinants	52	
		2.1.2	Condition of nonlinear maps	55	
		2.1.3	The moment maps \boldsymbol{G}_n and \boldsymbol{K}_n	57	
		2.1.4	Condition of $\boldsymbol{G}_n:\ \boldsymbol{\mu}\mapsto \boldsymbol{\gamma}$	59	
		2.1.5	$\text{Condition of } \boldsymbol{G}_n: \ \boldsymbol{m} \mapsto \boldsymbol{\gamma}$	64	
		2.1.6	Condition of $\boldsymbol{K}_n:\ \boldsymbol{m}\mapsto \boldsymbol{ ho}$	70	
		2.1.7	Modified Chebyshev algorithm	76	
		2.1.8	Finite expansions in orthogonal polynomials	78	
		2.1.9	Examples	82	
	2.2	Discr	etization methods	90	
		2.2.1	Convergence of discrete orthogonal polynomials		
			to continuous ones	90	
		2.2.2	A general-purpose discretization procedure	93	
		2.2.3	Computing the recursion coefficients of a		
			discrete measure	95	
		2.2.4	A multiple-component discretization method	99	
		2.2.5	Examples	101	
		2.2.6	Discretized modified Chebyshev algorithm	111	
	2.3	Comp	puting Cauchy integrals of orthogonal		
		polyn	nomials	112	
		2.3.1	Characterization in terms of minimal solutions	112	
		2.3.2	A continued fraction algorithm	113	
		2.3.3	Examples	116	
	2.4	Modi	fication algorithms	121	
		2.4.1	Christoffel and generalized Christoffel theorems	122	
		2.4.2	Linear factors	124	
		2.4.3	Quadratic factors	125	
		2.4.4	Linear divisors	128	
		2.4.5	Quadratic divisors	130	
		2.4.6	Examples	133	
	2.5	Comp	puting Sobolev orthogonal polynomials	138	
		2.5.1	Algorithm based on moment information	139	
		2.5.2	Stieltjes-type algorithm	141	
		2.5.3	Zeros	143	
		2.5.4	Finite expansions in Sobolev orthogonal		
			polynomials	146	
	2.6	Notes	s to Chapter 2	148	
3	Applications				
	3.1	Quad	Irature	152	
		3.1.1	Computation of Gauss-type quadrature		
			formulae	152	

CONTENTS

	3.1.2	Gauss–Kronrod quadrature formulae and their	105
		computation	165
	3.1.3	Gauss–Turán quadrature formulae and their	
		computation	172
	3.1.4	Quadrature formulae based on rational	
		functions	180
	3.1.5	Cauchy principal value integrals	202
	3.1.6	Polynomials orthogonal on several intervals	207
	3.1.7	Quadrature estimation of matrix functionals	211
3.2	Least	squares approximation	216
	3.2.1	Classical least squares approximation	217
	3.2.2	Constrained least squares approximation	221
	3.2.3	Least squares approximation in Sobolev spaces	225
3.3	Mom	ent-preserving spline approximation	227
	3.3.1	Approximation on the positive real line	228
	3.3.2	Approximation on a compact interval	237
3.4	Slowl	y convergent series	239
	3.4.1	Series generated by a Laplace transform	240
	3.4.2	"Alternating" series generated by a Laplace	
		transform	245
	3.4.3	Series generated by the derivative of a Laplace	
		transform	246
	3.4.4	"Alternating" series generated by the derivative	
		of a Laplace transform	248
	3.4.5	Slowly convergent series occurring in plate	
		contact problems	249
3.5	Notes	s to Chapter 3	253
Bibliog	Bibliography		
Index			283

vii