

Yuri I. Manin

Gauge Field Theory and Complex Geometry

Translated from the Russian by
N. Koblitz and J. R. King

With an Appendix by Sergei Merkulov

Second Edition



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