

# Texts and Monographs in Physics

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*Series Editors:* R. Balian W. Beiglböck H. Grosse E. H. Lieb  
N. Reshetikhin H. Spohn W. Thirring

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Giovanni Gallavotti

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# Statistical Mechanics

A Short Treatise

With 21 Figures



Springer

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*A Daniela, sempre*

# Preface

This book is the end result of a long story that started with my involvement as Coordinator of the Statistical Mechanics section of the Italian Encyclopedia of Physics.

An Italian edition collecting several papers that I wrote for the Encyclopedia appeared in September 1995, with the permission of the Encyclopedia and the sponsorship of Consiglio Nazionale delle Ricerche (CNR-GNFM).

The present work is not a translation of the Italian version but it overlaps with it: an important part of it (Chaps. 1–3, 8) is still based on three articles written as entries for the *Enciclopedia della Fisica* (namely: “*Meccanica Statistica*”, “*Teoria degli Insiemi*” and “*Moto Browniano*”) which make up about 29% of the present book and, furthermore, it still contains (with little editing and updating) my old review article on phase transitions (Chap. 6, published in *La Rivista del Nuovo Cimento*). In translating the ideas into English, I introduced many revisions and changes of perspective as well as new material (while also suppressing some other material).

The aim was to provide an analysis, intentionally as nontechnical as I was able to make it, of many fundamental questions of statistical mechanics, about two centuries after its birth. Only in a very few places have I entered into really technical details, mainly on subjects that I should know rather well or that I consider particularly important (the convergence of the Kirkwood-Salsburg equations, the existence of the thermodynamic limit, the exact solution of the Ising model, and in part the exact solution of the six vertex models). The points of view expressed here were presented in innumerable lectures and talks mostly to my students in Roma during the last 25 years. They are not always “mainstream views”; but I am confident that they are not too far from the conventionally accepted “truth” and I do not consider it appropriate to list the differences from other treatments. I shall consider this book a success if it prompts comments (even if dictated by strong disagreement or dissatisfaction) on the (few) points that might be controversial. This would mean that the work has attained the goal of being noticed and of being worthy of criticism.

I hope that this work might be useful to students by bringing to their attention problems which, because of “*concreteness necessities*” (*i.e.* because

such matters seem *useless*, or sometimes simply because of *lack of time*), are usually neglected even in graduate courses.

This does not mean that I intend to encourage students to look at questions dealing with the foundations of physics. I rather believe that young students should *refrain* from such activities, which should, possibly, become a subject of investigation after gaining an experience that only active and advanced research can provide (or at least the attempt at pursuing it over many years). And in any event I hope that the contents and the arguments I have selected will convey my appreciation for studies on the foundations that keep a strong character of concreteness. I hope, in fact, that this book will be considered concrete and far from speculative.

Not that students should not develop their own *philosophical* beliefs about the problems of the area of physics that interests them. Although one should be aware that any philosophical belief on the foundations of physics (and science), no matter how clear and irrefutable it might appear to the person who developed it after long meditations and unending vigils, is very unlikely to look less than objectionable to any other person who is given a chance to think about it, it is nevertheless necessary, in order to grow original ideas or even to just perform work of good technical quality, to possess precise philosophical convictions on the *rerum natura*, provided one is always willing to start afresh, avoiding, above all, thinking one has finally reached the *truth, unique, unchangeable and objective* (into whose existence only *vain* hope can be laid).

I am grateful to the *Enciclopedia Italiana* for having stimulated the beginning and the realization of this work, by assigning me the task of coordinating the Statistical Mechanics papers. I want to stress that the financial and cultural support from the *Enciclopedia* have been of invaluable aid. The atmosphere created by the Editors and by my colleagues in the few rooms of their facilities stimulated me deeply. It is important to remark on the rather unusual editorial enterprise they led to: it was not immediately animated by the logic of profit that moves the scientific book industry which is very concerned, at the same time, to avoid possible costly risks.

I want to thank G. Alippi, G. Altarelli, P. Dominici and V. Cappelletti who made a first version in Italian possible, mainly containing the Encyclopedia articles, by allowing the collection and reproduction of the texts of which the Encyclopedia retains the rights. I am indebted to V. Cappelletti for granting permission to include here the three entries I wrote for the *Enciclopedia delle Scienze Fisiche* (which is now published). I also thank the Nuovo Cimento for allowing the use of the 1972 review paper on the Ising model.

I am indebted for critical comments on the various drafts of the work, in particular, to G. Gentile whose comments have been an essential contribution to the revision of the manuscript; I am also indebted to several colleagues: P. Carta, E. Järvenpää, N. Nottingham and, furthermore, M. Campanino,

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Roma, January 1999

*Giovanni Gallavotti*



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