Springer Tracts in Natural Philosophy

Volume 38

Springer Science+Business Media, LLC

Edited by C. Truesdell

Springer Tracts in Natural Philosophy

Vol. 1 Gundersen: Linearized Analysis of One-Dimensional Magnetohydrodynamic Flows (1964).

Vol. 2 Walter: Differential- und Integral-Ungleichungen und ihre Anwendung bei Abschatzungs- und Eindeutigkeitsproblemen (1964).

Vol. 3 Gaier: Konstruktive Methoden der Konformen Abbildung (1964).

Vol. 4 Meinardus: Approximation von Funktionen und Ihre Numerische Behandlung (1964).

Vol. 5 Coleman/Markovitz/Noll: Viscometric Flows of Non-Newtonian Fluids: Theory and Experiment (1966).

Vol. 6 Eckhaus: Studies in Non-Linear Stability Theory (1965).

Vol. 7 Leimanis: The General Problem of the Motion of Coupled Rigid Bodies About a Fixed Point (1965).

Vol. 8 Roseau: Vibrations Non Linéaires et Théorie de la Stabilité (1966).

Vol. 9 Brown: Magnetoelastic Interactions (1966).

Vol. 10 Bunge: Foundations of Physics (1967).

Vol. 11 Lavrentiev: Some Improperly Posed Problems of Mathematical Physics (1967).

Vol. 12 Kronmuller: Nachwirkung in Ferromagnetika (1968).

Vol. 13 Meinardus: Approximation of Functions: Theory and Numerical Methods (1967).

Vol. 14 Bell: The Physics of Large Deformation of Crystalline Solids (1968).

Vol. 15 Buchholz: The Confluent Hypergeometric Function with Special Emphasis on Its Applications (1969).

Vol. 16 Slepian: Mathematical Foundations of Network Analysis (1968).

Vol. 17 Gavalas: Nonlinear Differential Equations of Chemically Reacting Systems (1968).

Vol. 18 Martin: Introduction to the Theory of Bases (1969).

Vol. 19 Knops/Payne: Uniqueness Theorems in Linear Elasticity

Vol. 20 Edelen/Wilson: Relativity and the Question of Discretization in Astonomy (1970).

Vol. 21 McBride: Obtaining Generating Functions (1971).

Vol. 22 Day: The Thermodynamics of Simple Materials with Fading Memory (1972).

(continued after index)

Giovanni P. Galdi

An Introduction to the Mathematical Theory of the Navier-Stokes Equations

Volume I

Linearised Steady Problems
Revised Edition

Springer

Giovanni P. Galdi
Istituto di Ingegneria
Università di Ferrara
Via Scandiana 21
44100 Ferrara
Italy

Mathematics Subject Classification (1991): 35Q30, 76D05, 76N10

With one illustration.

Library of Congress Cataloging-in-Publication Data
Galdi, Giovanni P. (Giovanni Paolo), 1947-
An introduction to the mathematical theory of the Navier-Stokes equations.
ISBN 978-1-4757-3868-1 ISBN 978-1-4757-3866-7 (eBook)
DOI 10.1007/978-1-4757-3866-7
(Springer tracts in natural philosphy ; v. 38-39)
Includes bibliographical references and index.
Contents: v. 1. Linearized steady problems — v. 2. Nonlinear steady problems.
1. Navier-Stokes equations. I. Title. II. Series.
QA377.G225 1994 515'.353 93-30352

Printed on acid-free paper.

© 1994 by Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 1994
Softcover reprint of the hardcover 1st edition 1994

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC, except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Hal Henglein; manufacturing supervised by Vincent Scelta.
Photocomposed copy prepared from the author's LaTeX files.

9 8 7 6 5 4 3 2 (Corrected second printing, 1998)

ISBN 978-1-4757-3868-1

SPIN 10654071

For Gino, Elena

and

Marisa

Preface

Undoubtedly, the Navier-Stokes equations are of basic importance within the context of modern theory of partial differential equations. Although the range of their applicability to concrete problems has now been clearly recognised to be limited, as my dear friend and bright colleague K.R. Rajagopal has showed me by several examples during the past six years, the mathematical questions that remain open are of such a fascinating and challenging nature that analysts and applied mathematicians cannot help being attracted by them and trying to contribute to their resolution. Thus, it is not a coincidence that over the past ten years more than seventy significant research papers have appeared concerning the well-posedness of boundary and initial-boundary value problems.

In this monograph I shall perform a systematic and up-to-date investigation of the fundamental properties of the Navier-Stokes equations, including existence, uniqueness, and regularity of solutions and, whenever the region of flow is unbounded, of their spatial asymptotic behavior. I shall omit other relevant topics like boundary layer theory, stability, bifurcation, detailed analysis of the behavior for large times, and free-boundary problems, which are to be considered "advanced" ones. In this sense the present work should be regarded as "introductory" to the matter.

I have divided the subject into two main parts: the first deals with steady flow (boundary value problem) and the second relates to time-dependent flow (initial-boundary value problem). Each part is self-contained and basically is not dependent on the other one, except for the case when the behavior in time of perturbations to a given steady motion will be investigated. In both parts the nonlinear analysis is always preceded and supported by a suitable linear analysis. This latter study, however, assumes an independent interest, and it is therefore treated in depth.

The *cantus firmus* of the entire work is represented by the approach in Lebesgue spaces L^q. In fact, this treatment is at the basis of the resolution of every problem considered throughout. I preferred to use such a general approach (instead of that in L^2 or weighted L^2), since it allows for a simpler and unified resolution of questions both in bounded and unbounded regions.

The book is essentially mathematically self-contained: the knowledge of Banach spaces and their basic properties (completeness, separability, reflexivity) along with some classical results on operator theory (*e.g.*, contraction mapping theorem) are the only necessary prerequisites to reading this book, which is devoted to students (graduate and undergraduate) and those mathematicians and applied mathematicians who wish to become acquainted with the subject.

I would like to thank warmly all my colleagues, co-workers, and friends who, directly or indirectly, have contributed to the realisation of this book.

First of all, I am deeply indebted to Clifford Truesdell who in September 1983 invited me to write this monograph for the series *Springer Tracts in Natural Philosophy* and who, over the past several years, has constantly encouraged my efforts.

Dan Joseph, Salvatore Rionero, and Jim Serrin have independently and affectionately guided my early steps on the hard and nonetheless fascinating path of the Navier-Stokes theory, each one teaching me different perspectives. To all of them I am grateful.

I express my sincere gratitude to Christian Simader and Hermann Sohr, with whom I have enjoyed working and casting new light on more than one problem, and to Paolo Maremonti, Wolfgang Borchers, and Konstantin Pileckas, who are coauthors with me of several papers.

I also acknowledge, with pleasure, many stimulating and helpful conversations I had with V. Coscia, R. Farwig, V.N. Maslennikova, A. Novotný, M.C. Patria, R. Rautmann, R. Salvi, V.A. Solonnikov, W. von Wahl, and M. Wiegner. In particular, I wish to thank V. Coscia, A. Novotný, A. Passerini, and M.C. Patria for reading parts of the manuscript and pointing out to me various misprints and mistakes.

I have received great benefit from teaching a course at the winter school in Paseky (Czechoslovakia) during the month of December 1991, and having discussions with well-trained young mathematicians and their trainer, my friend Professor Enrico Nečas. For this (at least), I wish to thank the organisers of the school and, in particular, Josef Málek (Donna Peppa) and Michael Růžička.

Last, but not least, I wish to acknowledge the generous help of my dear friend Dr. Gino Valenti, Director of the Construction Management Division in the Office of the Architect of the Capitol, for allowing me access to the Library of Congress in Washington, D.C., during the academic year 1984-1985.

At this point, a married author usually recognises the unselfish and continuous encouragement of his wife. In fact, it is for me a great privilege and pleasure to thank my wife, Professor Mariarosaria Padula, for spending endless and enjoyable (to me) days and nights discussing and setting up every topic of the book. To her, who in every respect has to be considered coauthor of this monograph, I offer my everlasting gratitude and love.

Ferrara, via Porta Romana, Giovanni Paolo Galdi
June 1992

Preface to the First Revised Edition

In this revised edition, I have corrected many misprints and some mistakes that were present in the first edition of the book. I wish to thank all colleagues who have kindly pointed them out to me. The list of their names is so long that if I would include it here, I would increase the probability of generating many other new misprints and mistakes, thus going through a neverending process.

There have been significant contributions to the mathematical theory of steady-state Navier-Stokes equations since the first edition of this book, like, for instance, those of K. Pileckas on the flow in domains with non-compact boundaries. These new achievements are reflected in the extended bibliography of this edition and referenced in the text. I have also entirely modified Section X.5 and, partly, Section X.6 to include the joint results of H. Sohr and me concerning the asymptotic structure of two-dimensional flow in exterior doimains.

I completed this work during the academic year 1996/1997, while I was visiting the Department of Mathematics at the University of Pittsburgh. I take this opportunity to thank my colleagues for warm hospitality and stimulating discussions and, in particular, John Chadam, Bill Layton and Bryce McLeod.

Pittsburgh, Beechwood Blvd,
April 1997

Giovanni Paolo Galdi

Contents