

Steven A. Gaal

Linear Analysis
and Representation
Theory



Springer-Verlag
New York Heidelberg Berlin 1973

Steven A. Gaal
University of Minnesota, School of Mathematics,
Minneapolis, Minnesota 55455, U.S.A.

Geschäftsführende
Herausgeber

B. Eckmann
Eidgenössische Technische Hochschule Zürich

B. L. van der Waerden
Mathematisches Institut der Universität Zürich

AMS Subject Classifications (1970)

Primary: 22 D 05, 22 D 10, 22 D 15, 22 D 25, 22 D 30, 22 E 15,
22 E 60, 28 A 70, 43 A 35, 43 A 65, 43 A 85, 43 A 90

Secondary: 46 H 05, 46 J 05, 47 A 10, 47 B 05, 47 B 10, 58 A 05

ISBN-13:978-3-642-80743-5

e-ISBN-13:978-3-642-80741-1

DOI: 10.1007/978-3-642-80741-1

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher. © by Springer-Verlag Berlin Heidelberg 1973. Library of Congress Catalog Card Number 72-95686.
Softcover reprint of the hardcover 1st edition 1973

Table of Contents

Chapter I. Algebras and Banach Algebras	1
1. Algebras and Norms	1
2. The Group of Units and the Quasigroup	5
3. The Maximal Ideal Space.	8
4. The Spectrum of an Element	10
5. The Spectral Norm Formula	13
6. Commutative Banach Algebras and their Ideals	16
7. Radical and Semisimplicity	26
8. Involution Algebras	37
9. H^* Algebras	45
Remarks	52
Chapter II. Operators and Operator Algebras	55
1. Topologies on Vector Spaces and on Operator Algebras	55
2. Compact Operators	65
3. The Spectral Theorem for Compact Operators	68
4. Hilbert-Schmidt Operators	71
5. Trace Class Operators	76
6. Vector Valued Line Integrals	82
7. Homomorphisms into A . The Spectral Mapping Theorem	85
8. Unbounded Operators	90
Remarks	99
Chapter III. The Spectral Theorem, Stable Subspaces and v. Neumann Algebras	102
1. Linear Functionals on Vector Lattices and their Extensions	102
2. Linear Functionals on Lattices of Functions.	108
3. The Spectral Theorem for Self Adjoint Operators in Hilbert Space	111
4. Normal Elements and Normal Operators	116
5. Stable Subspaces and Commutants	122
6. von Neumann Algebras	127
7. Measures on Locally Compact Spaces	135
Remarks	143

Chapter IV. Elementary Representation Theory in Hilbert Space	145
1. Representations and Morphisms.	145
2. Irreducible Components, Equivalence	150
3. Intertwining Operators	158
4. Schur's Lemma	161
5. Multiplicity of Irreducible Components.	168
6. The General Trace Formula	175
7. Primary Representations and Factorial v. Neumann Algebras	183
8. Algebras and Representations of Type I	198
9. Type II and III v. Neumann Algebras	211
Remarks	225
Preliminary Remarks to Chapter V.	227
Chapter V. Topological Groups, Invariant Measures, Convolutions and Representations	228
1. Topological Groups and Homogeneous Spaces	228
2. Haar Measure	241
3. Quasi-Invariant and Relatively Invariant Measures.	257
4. Convolutions of Functions and Measures	270
5. The Algebra Representation Associated with $\rho: S \rightarrow \mathcal{L}(\mathcal{H})$	284
6. The Regular Representations of Locally Compact Groups.	299
7. Continuity of Group Representations and the Gelfand-Raikov Theorem	303
Remarks	318
Chapter VI. Induced Representations	321
1. The Riesz-Fischer Theorem	321
2. Induced Representations when G/H has an Invariant Measure	325
3. Tensor Products	332
4. Induced Representations for Arbitrary G and H	347
5. The Existence of a Kernel for $\sigma: L^1(G) \rightarrow \mathcal{L}(\mathcal{H})$	360
6. The Direct Sum Decomposition of the Induced Representation $\rho^\lambda: G \rightarrow \mathcal{U}(\mathcal{K})$	367
7. The Isometric Isomorphism between \mathcal{L}^2 and $HS(\mathcal{K}_2, \mathcal{K}_1)$. The Computation of the Trace in Terms of the Associated Kernel	376
8. The Tensor Product of Induced Representations	387
9. The Theorem on Induction in Stages	393
10. Representations Induced by Representations of Conjugate Sub- groups	398
11. Mackey's Theorem on Strong Intertwining Numbers and Some of its Consequences	402

12. Isomorphism Theorems Implying the Frobenius Reciprocity Relation	411
Remarks	419
 Chapter VII. Square Integrable Representations, Spherical Functions and Trace Formulas	 423
1. Square Integrable Representations and the Representation Theory of Compact Groups	423
2. Zonal Spherical Functions	444
3. Spherical Functions of Arbitrary Type and Height	460
4. Godement's Theorem on the Characterization of Spherical Functions	477
5. Representations of Groups with an Iwasawa Decomposition.	494
6. Trace Formulas	510
Remarks	526
 Chapter VIII. Lie Algebras, Manifolds and Lie Groups	 531
1. Lie Algebras	531
2. Finite Dimensional Representations of Lie Algebras. Cartan's Criteria and the Theorems of Engel and Lie	546
3. Presheaves and Sheaves	560
4. Differentiable Manifolds	573
5. Lie Groups and their Lie Algebras	588
6. The Exponential Map and Canonical Coordinates	608
7. Lie Subgroups and Subalgebras	629
8. Invariant Lie Subgroups and Quotients of Lie Groups. The Projective Groups and the Lorentz Group	644
Remarks	659
Bibliography	663
Subject Index	672
Index of Notations and Special Symbols	682