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Fundamentals of Diophantine Geometry

Springer-Verlag

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AMS Subject Classifications: 10B99, 14GXX

Library of Congress Cataloging in Publication Data
Lang, Serge, 1927–
Fundamentals of diophantine geometry.
1. Diophantine analysis. 2. Geometry, Algebraic.
I. Title.
QA242.L235 1983 512′.74 83-361

An earlier version of this book, *Diophantine Geometry*, was published by Wiley-Interscience.

ISBN 978-1-4419-2818-4 ISBN 978-1-4757-1810-2 (eBook)
DOI 10.1007/978-1-4757-1810-2

Published in 1983 by Springer Science+Business Media New York
Originally published by Springer-Verlag New York Inc in 1983.
Softcover reprint of the hardcover 1st edition 1983

Typeset by Composition House Ltd., Salisbury, England.

9 8 7 6 5 4 3 2 1

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Acknowledgment

I thank Michel Laurent and Michel Waldschmidt for useful comments. I am especially indebted to Joe Silverman for his thorough going over of the manuscript and a long list of valuable suggestions and corrections. I am also indebted to Silverman and Tate for their manuscripts which formed the basis for the next to last chapter.

Some Standard Notation

The following notation is used in a standard way throughout.

Rings are assumed commutative and without divisors of 0, unless otherwise specified.

$\boldsymbol{\mu}$	group of all roots of unity.
$\boldsymbol{\mu}(K)$	subgroup of roots of unity in a field K.
R^*, K^*	invertible elements in a ring R (resp. in a field K).
K^{a}	algebraic closure of a field K.
$K(P)$	field obtained by adjoining to K a set of affine coordinates for a point P (equal to $K(x_0/x_i, \ldots, x_n/x_i)$ if $(x_0, \ldots, x_n)$ are projective coordinates for P).
$R(\mathfrak{a})$	$R/\mathfrak{a}$ for any ideal $\mathfrak{a}$.
$\mathbf{Z}(N)$	$\mathbf{Z}/N\mathbf{Z}$.
$A^{(p)}$	p-primary part of an abelian group A (that is, the subgroup of elements whose order is a power of p).
A_m	subgroup of elements x in an abelian group A such that $mx = 0$.
$[n]$	multiplication by an integer n on an abelian group.
$V(K)$	set of K-valued points of a variety or scheme V.
$D \sim D'$	for divisors D, D', linear equivalence.
$D \approx D'$	for divisors D, D', algebraic equivalence.
$\mathrm{Div}(V)$	group of divisors on a variety V.
$\mathrm{Div}_a(V)$	subgroup of divisors algebraically equivalent to 0.
$\mathrm{Div}_l(V)$	subgroup of divisors linearly equivalent to 0.

$\mathrm{Pic}(V)$	$\mathrm{Div}(V)/\mathrm{Div}_l(V)$.
$\mathrm{Pic}_0(V)$	$\mathrm{Div}_a(V)/\mathrm{Div}_l(V)$.
$\mathrm{NS}(V)$	$\mathrm{Div}(V)/\mathrm{Div}_a(V)$ (the **Néron–Severi** group of V).
$h \sim h'$	equivalence for functions, $\lvert h - h' \rvert$ is bounded.
$h \approx h'$	quasi-equivalence for functions: for each $\varepsilon > 0$, $-C_1 + (1 - \varepsilon)h \leqq h' \leqq (1 + \varepsilon)h + C_2$.
$h \ll h'$	for functions, with h' positive, there exists a constant $C > 0$ such that $\lvert h \rvert \leqq Ch'$. Same as $h = O(h')$.
$h \gg\ll h'$	both h, h' positive, $h \ll h'$ and $h' \ll h$.

Usually h, H denote **heights** with $h = \log H$. These are indexed to specify qualifications:

h_φ	height determined by a morphism φ into projective space.
k_K	height relative to a field K.
h_X	height determined by a morphism derived from the linear system $\mathscr{L}(X)$, well defined up to $O(1)$.
h_c	on an arbitrary variety, the height associated with a divisor class c, determined only up to $O(1)$; on an abelian variety, the canonical height.
$\hat{h}_c$	canonical height if one needs to distinguish it from an equivalence class of heights.

Standard references:

IAG = *Introduction to Algebraic Geometry* [L 2].

AV = *Abelian Varieties* [L 3].

Weil's *Foundations* is still quoted in canonical style, F^2–X_y, Theorem Z, which refers to the second edition.

For schemes, see Hartshorne's *Algebraic Geometry*, and also Mumford's *Abelian Varieties*.