Intersection Theory

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Second Edition

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Preface to the Second Edition

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No attempt has been made to survey the many developments in intersection theory since 1983, other than adding some references which appeared not long after first edition. A few indications to more recent literature, as well as an informal introduction to the main ideas of this book, can be found in the 1996 edition of the author's *Introduction to Intersection Theory in Algebraic Geometry*, CBMS 57, Amer. Math. Soc., 1984, 1996.

October, 1997 *William Fulton*

Preface to the First Edition

From the ancient origins of algebraic geometry in the solution of polynomial equations, through the triumphs of algebraic geometry during the last two centuries, intersection theory has played a central role. Since its role in foundational crises has been no less prominent, the lack of a complete modern treatise on intersection theory has been something of an embarrassment. The aim of this book is to develop the foundations of intersection theory, and to indicate the range of classical and modern applications. Although a comprehensive history of this vast subject is not attempted, we have tried to point out some of the striking early appearances of the ideas of intersection theory.

Recent improvements in our understanding not only yield a stronger and more useful theory than previously available, but also make it possible to develop the subject from the beginning with fewer prerequisites from algebra and algebraic geometry. It is hoped that the basic text can be read by one equipped with a first course in algebraic geometry, with occasional use of the two appendices. Some of the examples, and a few of the later sections, require more specialized knowledge. The text is designed so that one who understands the constructions and grants the main theorems of the first six chapters can read other chapters separately. Frequent parenthetical references to previous sections are included for such readers. The summaries which begin each chapter should facilitate use as a reference.

Several theorems are new or stronger than those which have appeared before, and some proofs are significantly simpler. Among the former are a new blow-up formula, a stronger residual intersection formula, and the removal of a projective hypotheses from intersection theory and Riemann-Roch theorems; the latter includes the proof of the Grothendieck-Riemann-Roch theorem. Some formulas from classical enumerative geometry receive a first modern or rigorous proof here.

Acknowledgements. The intersection theory described here was developed together with R. MacPherson. The author whose name appears on the cover is responsible for the presentation of details, and many of the applications and examples, but the extent to which it forms a coherent theory derives from collaboration with MacPherson. Previously unpublished results of R. Lazarsfeld, and joint work with Lazarsfeld, and with H. Gillet, is also included. During the course of the work, many helpful suggestions were made by A. Collino, P. Deligne, S. Diaz, J. Harris, B. Iversen, S. L. Kleiman, A. Landman, Lazarsfeld, and J-P. Serre. Although other contributions and historical precedents are acknowledged in the text, many others, such as those of students and others who have responded to talks on these subjects, must be silently, but gratefully, cited.

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