Undergraduate Texts in Mathematics

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Optimization Techniques
An Introduction

With 72 Illustrations

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This book is dedicated to the memory of my father

Richard Seddon Foulds

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Preface

Optimization is the process by which the optimal solution to a problem, or optimum, is produced. The word *optimum* has come from the Latin word *optimus*, meaning best. And since the beginning of his existence Man has strived for that which is best. There has been a host of contributions, from Archimedes to the present day, scattered across many disciplines. Many of the earlier ideas, although interesting from a theoretical point of view, were originally of little practical use, as they involved a daunting amount of computational effort. Now modern computers perform calculations, whose time was once estimated in man-years, in the figurative blink of an eye. Thus it has been worthwhile to resurrect many of these earlier methods. The advent of the computer has helped bring about the unification of optimization theory into a rapidly growing branch of applied mathematics. The major objective of this book is to provide an introduction to the main optimization techniques which are at present in use. It has been written for final year undergraduates or first year graduates studying mathematics, engineering, business, or the physical or social sciences. The book does not assume much mathematical knowledge. It has an appendix containing the necessary linear algebra and basic calculus, making it virtually self-contained.

This text evolved out of the experience of teaching the material to finishing undergraduates and beginning graduates. A feature of the book is that it adopts the sound pedagogical principle that an instructor should proceed from the known to the unknown. Hence many of the ideas in the earlier chapters are introduced by means of a concrete numerical example to which the student can readily relate. This is followed by generalization to the underlying theory. The courses on which the book is based usually have a significant number of students of Business and Engineering. The interests

of these people have been taken into account in the development of the courses and hence in the writing of this book. Hence many of its arguments are intuitive rather than rigorous. Indeed plausibility and clarity have been given precedence before rigour for the sake of itself.

Chapter 1 contains a brief historical account and introduces the basic terminology and concepts common to all the theory of optimization. Chapters 2 and 3 are concerned with linear programming and complications of the basic model. Chapter 2 on the simplex method, duality, and sensitivity analysis can be covered in an undergraduate course. However some of the topics in Chapter 3 such as considerations of efficiency and parametric programming, may be best left to graduate level. Chapter 4 deals with only the basic strategies of integer linear programming. It is of course dependent on Chapter 2. It does contain a number of formulations of applications of integer programming. Some of this material has never appeared before in book form. Chapter 5 is on network analysis and contains a section on using networks to analyze some practical problems.

Chapter 6 introduces dynamic programming. It is beyond the scope of this book to provide a detailed account of this vast topic. Hence techniques suitable for only deterministic, serial systems are presented. The interested reader is referred to the extensive literature. Chapter 7 serves as an introduction to Chapter 8, which is on nonlinear programming. It presents some of the classical techniques: Jacobian and Lagrangian methods together with the Kuhn–Tucker conditions. The ideas in this chapter are used in devising the more computationally efficient strategies of Chapter 8.

This text contains enough material for one semester at the undergraduate level and one more at the graduate level. The first course could contain Chapters 1, 2, the first half of Chapter 3, and parts of Chapter 4 and Chapter 5. The remainder can be covered in the second course. A plan outlining this follows.

The book contains a large number of exercises. Students are strongly encouraged to attempt them. One cannot come to grips with the concepts by solely looking at the work of others. Mathematics is not a spectator sport!

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Christchurch, New Zealand L. R. Foulds
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Plan of the Book