Undergraduate Texts in Mathematics

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continued after Index

L. R. Foulds

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With 56 Illustrations

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L. R. Foulds
Department of Industrial and Systems Engineering
University of Florida
Gainesville, FL 32611
U.S.A.

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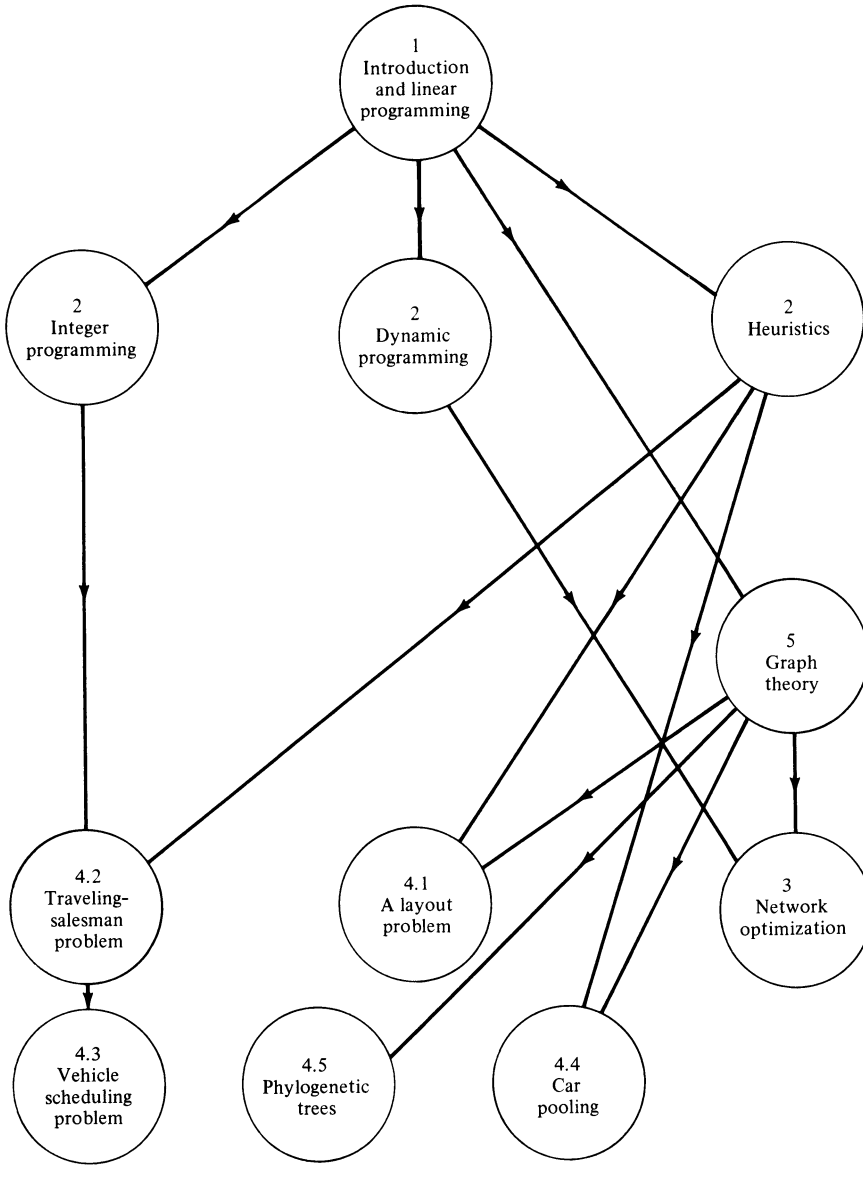
To my parents, Edith and Reginald,
of Ruakaka, New Zealand

Preface

The major purpose of this book is to introduce the main concepts of discrete optimization problems which have a finite number of feasible solutions. Following common practice, we term this topic *combinatorial optimization*. There are now a number of excellent graduate-level textbooks on combinatorial optimization. However, there does not seem to exist an undergraduate text in this area. This book is designed to fill this need.

The book is intended for undergraduates in mathematics, engineering, business, or the physical or social sciences. It may also be useful as a reference text for practising engineers and scientists. The writing of this book was inspired through the experience of the author in teaching the material to undergraduate students in operations research, engineering, business, and mathematics at the University of Canterbury, New Zealand. This experience has confirmed the suspicion that it is often wise to adopt the following approach when teaching material of the nature contained in this book. When introducing a new topic, begin with a numerical problem which the students can readily understand; develop a solution technique by using it on this problem; then go on to general problems. This philosophy has been adopted throughout the book. The emphasis is on plausibility and clarity rather than rigor, although rigorous arguments have been used when they contribute to the understanding of the mechanics of an algorithm. An example of this is furnished by the construction of the labeling method for the maximal-network-flow problem from the proof of the max-flow, min-cut theorem.

The book comprises two parts—Part I: Techniques and Part II: Applications. Part I begins with a motivational chapter which includes a description of the general combinatorial optimization problem, important current problems, a description of the fundamental algorithm, a discussion of the



Plan of the Book.

need for efficient algorithms, and the effect of the advent of the digital computer. This is followed by a chapter on linear programming and its extensions. Chapter 2 describes the basic procedures of three of the most important combinatorial optimization techniques—integer programming, dynamic programming, and heuristic methods. Chapter 3 is concerned with optimization on graphs and networks.

Part II poses a variety of problems from many different disciplines—the traveling-salesman problem, the vehicle scheduling problem, car pooling,

evolutionary tree construction, and the facilities layout problem. Each problem is analyzed and solution procedures are then presented. Some of these procedures have never appeared before in book form.

The book contains a number of exercises which the reader is strongly encouraged to try. Mathematics is not a spectator sport! These exercises range from routine numerical drill-type exercises to open questions from the research literature. The more challenging problems have an asterisk preceding them. The author is grateful for this opportunity to express his thanks for the support he received from the University of Canterbury while writing this book, and to his doctoral student John Giffin, who contributed to Section 4.1. He is also extremely thankful to his wife Maureen, who not only provided enthusiastic encouragement, but also typed the complete manuscript. Finally, the author pays a hearty tribute to the staff at Springer-Verlag New York for their patience, skill, and cooperation during the preparation of this book.

Gainesville, Florida L. R. FOULDS

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