

# CONTENTS

PREFACE. . . . .	PAGE v
------------------	-----------

## CHAPTER I

### LINEAR TRANSFORMATIONS

1. The Linear Transformation . . . . .	1
2. Symbolic Notation . . . . .	4
3. The Fixed Points of the Transformation. . . . .	6
4. The Linear Transformation and the Circle. . . . .	8
5. Inversion in a Circle . . . . .	10
6. The Multiplier, $K$ . . . . .	15
7. The Hyperbolic Transformation, $K = A$ . . . . .	18
8. The Elliptic Transformation, $K = e^{i\theta}$ . . . . .	19
9. The Loxodromic Transformation, $K = Ae^{i\theta}$ . . . . .	20
10. The Parabolic Transformation. . . . .	21
11. The Isometric Circle . . . . .	23
12. The Unit Circle . . . . .	30

## CHAPTER II

### GROUPS OF LINEAR TRANSFORMATIONS

13. Definition of a Group. Examples . . . . .	33
14. Properly Discontinuous Groups . . . . .	35
15. Transforming a Group . . . . .	36
16. The Fundamental Region. . . . .	37
17. The Isometric Circles of a Group. . . . .	39
18. The Limit Points of a Group. . . . .	41
19. Definition of the Region $R$ . . . . .	44
20. The Regions Congruent to $R$ . . . . .	44
21. The Boundary of $R$ . . . . .	47
22. Example. A Finite Group . . . . .	49
23. Generating Transformations. . . . .	50
24. Cyclic Groups. . . . .	51
25. The Formation of Groups by the Method of Combination. . . . .	56
26. Ordinary Cycles . . . . .	59
27. Parabolic Cycles. . . . .	62
28. Function Groups. . . . .	64

## CHAPTER III

### FUCHSIAN GROUPS

29. The Transformations. . . . .	67
30. The Limit Points. . . . .	67

# CONTENTS

	PAGE
31. The Region $R$ and the Region $R_o$ .	69
32. Generating Transformations.	71
33. The Cycles	72
34. Fuchsian Groups of the First and Second Kinds	73
35. Fixed Points at Infinity. Extension of the Method	75
36. Examples.	78
37. The Modular Group	79
38. Some Subgroups of the Modular Group.	81

## CHAPTER IV

### AUTOMORPHIC FUNCTIONS

39. The Concept of the Automorphic Function	83
40. Simple Automorphic Functions.	86
41. Behavior at Vertices and Parabolic Points.	88
42. The Poles and Zeros	91
43. Algebraic Relations.	94
44. Differential Equations	98

## CHAPTER V

### THE POINCARÉ THETA SERIES

45. The Theta Series.	102
46. The Convergence of the Series.	104
47. The Convergence for the Fuchsian Group of the Second Kind	106
48. Some Properties of the Theta Functions.	108
49. Zeros and Poles of the Theta Functions.	112
50. Series and Products Connected with the Group	115

## CHAPTER VI

### THE ELEMENTARY GROUPS

#### I. The Finite Groups

51. Inversion in a Sphere.	117
52. Stereographic Projection	119
53. Rotations of the Sphere.	120
54. Groups of the Regular Solids	123
55. A Study of the Cube	124
56. The General Regular Solid	127
57. Determination of All the Finite Groups.	129
58. The Extended Groups	136

#### II. The Groups with One Limit Point

59. The Simply and Doubly Periodic Groups	139
60. Groups Allied to the Periodic Groups.	140
61. The Automorphic Functions.	144

#### III. The Groups with Two Limit Points

62. Determination of the Groups	146
---------------------------------	-----

# CONTENTS

## CHAPTER VII

### THE ELLIPTIC MODULAR FUNCTIONS

	PAGE
63. Certain Results from the Theory of Elliptic Functions . . . . .	148
64. Change of the Primitive Periods . . . . .	150
65. The Function $J(\tau)$ . . . . .	151
66. Behavior of $J(\tau)$ at the Parabolic Points . . . . .	153
67. Further Properties of $J(\tau)$ . . . . .	155
68. The Function $\lambda(\tau)$ . . . . .	157
69. The Relation between $\lambda(\tau)$ and $J(\tau)$ . . . . .	159
70. Further Properties of $\lambda(\tau)$ . . . . .	160

## CHAPTER VIII

### CONFORMAL MAPPING

71. Conformal Mapping . . . . .	164
72. Schwarz's Lemma . . . . .	165
73. Area Theorems. . . . .	167
74. The Mapping of a Circle on a Plane Finite Region. . . . .	169
75. The Deformation Theorem for the Circle . . . . .	171
76. A General Deformation Theorem. . . . .	175
77. An Application of Poisson's Integral . . . . .	177
78. The Mapping of a Plane Simply Connected Region on a Circle. The Iterative Process. . . . .	179
79. The Convergence of the Process . . . . .	183
80. The Behavior of the Mapping Function on the Boundary. . . . .	187
81. Regions Bounded by Jordan Curves . . . . .	198
82. Analytic Arcs and the Continuation of the Mapping Function across the Boundary . . . . .	201
83. Circular Arc Boundaries. . . . .	202
84. The Mapping of Combined Regions . . . . .	203
85. The Mapping of Limit Regions. . . . .	205
86. The Mapping of Simply Connected Finite-sheeted Regions . . . . .	213
87. Conformal Mapping and Groups of Linear Transformations . . . . .	216

## CHAPTER IX

### UNIFORMIZATION. ELEMENTARY AND FUCHSIAN FUNCTIONS

88. The Concept of Uniformization . . . . .	220
89. The Connectivity of Regions. . . . .	221
90. Algebraic Functions of Genus Zero. Uniformization by Means of Rational Functions. . . . .	229
91. Algebraic Functions of Genus Greater than Zero. Uniformiza- tion by Means of Automorphic Functions. . . . .	233
92. The Genus of the Fundamental Region of a Group. . . . .	238
93. The Cases $p = 1$ and $p > 1$ . . . . .	239
94. More General Fuchsian Uniformizing Functions . . . . .	241
95. The Case $p = 0$ . . . . .	245

# CONTENTS

	PAGE
96. Whittaker's Groups. . . . .	247
97. The Transcendental Functions. . . . .	249

## CHAPTER X

### UNIFORMIZATION. GROUPS OF SCHOTTKY TYPE

98. Regions of Planar Character. . . . .	256
99. Some Accessory Functions. . . . .	258
100. The Mapping of a Multiply Connected Region of Planar Character on a Slit Region. . . . .	262
101. Application to the Uniformization of Algebraic Functions. . . . .	266
102. A Convergence Theorem . . . . .	267
103. The Sequence of Mapping Functions . . . . .	271
104. The Linearity of $T_n$ . . . . .	273
105. An Extension . . . . .	278
106. The Mapping of a Multiply Connected Region of Planar Character on a Region Bounded by Complete Circles. . . . .	279

## CHAPTER XI

### DIFFERENTIAL EQUATIONS

107. Connection with Groups of Linear Transformations. . . . .	284
108. The Inverse of the Quotient of Two Solutions . . . . .	287
109. Regular Singular Points of Differential Equations . . . . .	293
110. The Quotient of Two Solutions at a Regular Singular Point . . . . .	296
111. Equations with Rational Coefficients . . . . .	299
112. The Equation with Two Singular Points . . . . .	303
113. The Hypergeometric Equation. . . . .	303
114. The Riemann-Schwarz Triangle Functions. . . . .	305
115. Equations with Algebraic Coefficients. . . . .	308

A BIBLIOGRAPHY OF AUTOMORPHIC FUNCTIONS . . . . .	311
---	-----

AUTHOR INDEX. . . . .	325
-----------------------	-----

SUBJECT INDEX . . . . .	327
-------------------------	-----