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Topological Modeling for Visualization

With 337 Figures, Including 7 in Color



Springer

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ISBN 978-4-431-66958-6 ISBN 978-4-431-66956-2 (eBook)
DOI 10.1007/978-4-431-66956-2

Printed on acid-free paper

© Springer Japan 1997

Originally published by Springer-Verlag Tokyo Berlin Heidelberg New York in 1997

Softcover reprint of the hardcover 1st edition 1997

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Typesetting: Camera-ready by authors

Preface

The flood of information through various computer networks such as the Internet characterizes the world situation in which we live. Information worlds, often called virtual spaces and cyberspaces, have been formed on computer networks. The complexity of information worlds has been increasing almost exponentially through the exponential growth of computer networks. Such nonlinearity in growth and in scope characterizes information worlds. In other words, the characterization of nonlinearity is the key to understanding, utilizing and living with the flood of information. The characterization approach is by characteristic points such as peaks, pits, and passes, according to the Morse theory. Another approach is by singularity signs such as folds and cusps. Atoms and molecules are the other fundamental characterization approach. Topology and geometry, including differential topology, serve as the framework for the characterization. *Topological Modeling for Visualization* is a textbook for those interested in this characterization, to understand what it is and how to do it. Understanding is the key to utilizing information worlds and to living with the changes in the real world.

Writing this textbook required careful preparation by the authors. There are complex mathematical concepts that require designing a writing style that facilitates understanding and appeals to the reader. To evolve a style, we set as a main goal of this book the establishment of a link between the theoretical aspects of modern geometry and topology, on the one hand, and experimental computer geometry, on the other. There are many excellent books on modern geometry and topology (generally speaking, “theory”), and many excellent books on modern computer and experimental geometry. As far as we know, however, there is no book that bridges the gap between these two branches of modern science, that is, between theory and practice. We have tried to fill this gap. Our intention was to write a book that will be useful to both communities of scientists. Of course, we realize that this separation between theoretical science and experimental science is not clear-cut, and we use this language and these images only for easier description of our main idea. We collect in the book some basic elements of theoretical geometry and topology that are used today in different branches of experimental computer geometry. We do not give detailed proofs because of lack of space, but we give references that can help the reader find the proofs. The advantage of such a style is this:

We collect in one book a short description of the most powerful theoretical tools, and experts in experimental science can use this material in their work. Certainly, as we know from our own experience, modern topological methods can improve the results of experimental computer geometry. On the other hand, experts in theoretical geometry and topology can find in our book possible applications of those fields to very interesting computer experiments in the world of geometrical computer methods, medicine, the automobile industry, architecture, and so on. Many pure mathematicians will also find here material for development of new theoretical ideas. Each chapter consists of two layers: first theoretical ideas, then applications to the different branches of modern experimental computer geometry. We have tried to make chapters as independent as possible to help the reader use each chapter as an individual research tool without a complete study of other sections of the book. Consequently, we sometimes include in some chapters a summary of material from another section to recall important ideas.

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Acknowledgments. The project to publish a textbook on topological modeling for visualizing complex objects and phenomena has been a long-standing desire of the authors. We now have the initial version. Many people have contributed to the project. Springer-Verlag has done an excellent job of improving the text and the style. Multiple sources of support have been helpful, including the MOVE Project sponsored by Fujitsu, Microsoft Japan, and Ricoh; the University of Aizu; the University of Tokyo; Moscow State University; and the Aizu Area Foundation for Promotion of Science and Education.

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