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2nd Edition 1991 · 5th Printing 2003

ISSN 0172-5726

ISBN 978-3-540-53601-7 ISBN 978-3-642-58239-4 (eBook)

DOI 10.1007/978-3-642-58239-4

Library of Congress Cataloging-in-Publication Data. Fletcher, C. A. J. Computational techniques for fluid dynamics 2 : specific techniques for different flow categories / C. A. J. Fletcher. – 2nd ed. p. cm. – (Springer series in computational physics) Includes bibliographical references and index. ISBN 978-3-540-53601-7 1. Fluid dynamics – Mathematics. 2. Fluid dynamics – Data processing. 3. Mathematical analysis. I. Title. II. Series. QC151.F58 1991 532'.05'0151 – dc20 91-7345

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Originally published by Springer-Verlag Berlin Heidelberg New York in 1991

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Cover design: *design & production* GmbH, Heidelberg

Typesetting: Macmillan India Ltd., India

55/3111 – 5 – Printed on acid-free paper

Preface to the Second Edition

The purpose and organisation of this book are described in the preface to the first edition (1988). In preparing this edition minor changes have been made, particularly to Chap. 1 (Vol. 1) to keep it reasonably current, and to upgrade the treatment of specific techniques, particularly in Chaps. 12–14 and 16–18. However, the rest of the book (Vols. 1 and 2) has required only minor modification to clarify the presentation and to modify or replace individual problems to make them more effective. The answers to the problems are available in *Solutions Manual for Computational Techniques for Fluid Dynamics* by K. Srinivas and C. A. J. Fletcher, published by Springer-Verlag, Heidelberg, 1991. The computer programs have also been reviewed and tidied up. These are available on an IBM-compatible floppy disc direct from the author.

I would like to take this opportunity to thank the many readers for their usually generous comments about the first edition and particularly those readers who went to the trouble of drawing specific errors to my attention. In this revised edition considerable effort has been made to remove a number of minor errors that had found their way into the original. I express the hope that no errors remain but welcome communication that will help me improve future editions.

In preparing this revised edition I have received considerable help from Dr. K. Srinivas, Nam-Hyo Cho, Zili Zhu and Susan Gonzales at the University of Sydney and from Professor W. Beiglböck and his colleagues at Springer-Verlag. I am very grateful to all of them.

Sydney, March 1991

C. A. J. Fletcher

Preface to the First Edition

As indicated in Vol. 1, the purpose of this two-volume textbook is to provide students of engineering, science and applied mathematics with the specific techniques, and the framework to develop skill in using them, that have proven effective in the various branches of computational fluid dynamics.

Volume 1 describes both fundamental and general techniques that are relevant to all branches of fluid flow. This volume contains specific techniques applicable to the different categories of engineering flow behaviour, many of which are also appropriate to convective heat transfer.

The contents of Vol. 2 are suitable for specialised graduate courses in the engineering computational fluid dynamics (CFD) area and are also aimed at the established research worker or practitioner who has already gained some fundamental CFD background. It is assumed that the reader is familiar with the contents of Vol. 1.

The contents of Vol. 2 are arranged in the following way: Chapter 11 develops and discusses the equations governing fluid flow and introduces the simpler flow categories for which specific computational techniques are considered in Chaps. 14–18.

Most practical problems involve computational domain boundaries that do not conveniently coincide with coordinate lines. Consequently, in Chap. 12 the governing equations are expressed in generalised curvilinear coordinates for use in arbitrary computational domains. The corresponding problem of generating an interior grid is considered in Chap. 13.

Computational techniques for inviscid flows are presented in Chap. 14 for incompressible, supersonic and transonic conditions. In Chap. 15 methods are described for predicting the flow behaviour in boundary layers.

For many steady flows with a dominant flow direction it is possible to obtain accurate flow predictions, based on reduced forms of the Navier-Stokes equations, in a very efficient manner. Such techniques are developed in Chap. 16. In Chaps. 17 and 18 specific computational methods are discussed for separated flows, governed by the incompressible and compressible Navier-Stokes equations respectively.

In preparing this textbook I have been assisted by many people, some of whom are acknowledged in the Preface of Vol. 1. However, the responsibility for any errors or omissions remaining rests with me. Any comments, criticism and suggestions that will improve this textbook are most welcome and will be gratefully received.

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