Fractal Image Compression

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Editor

Fractal Image Compression

Theory and Application

With 139 Illustrations

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Preface

What is "Fractal Image Compression," anyway? You will have to read the book to find out everything about it, and if you read the book, you really will find out almost everything that is currently known about it. In a sentence or two: fractal image compression is a method, or class of methods, that allows images to be stored on computers in much less memory than standard ways of storing images. The "fractal" part means that the methods have something to do with fractals, complicated looking sets that arise out of simple algorithms.

This book contains a collection of articles on fractal image compression. Beginners will find simple explanations, working C code, and exercises to check their progress. Mathematicians will find a rigorous and detailed development of the subject. Non-mathematicians will find a parallel intuitive discussion that explains what is behind all the "theorem–proofs." Finally, researchers – even researchers in fractal image compression – will find new and exciting results, both theoretical and applied.

Here is a brief synopsis of each chapter:

Chapter 1 contains a simple introduction aimed at the lay reader. It uses almost no math but explains all the main concepts of a fractal encoding/decoding scheme, so that the interested reader can write his or her own code.

Chapter 2 has a rigorous mathematical description of iterated function systems and their generalizations for image encoding. An informal presentation of the material is made in parallel in the chapter using sans serif font.

Chapter 3 contains a detailed description of a quadtree-based method for fractal encoding. The chapter is readily accessible, containing no mathematics. It does contain almost everything anyone would care to know about the quadtree method.

The following chapters are contributed articles.

Chapter 4 details an important optimization which can reduce encoding times significantly. It naturally follows the previous chapter, but the methods can be applied in more general settings.

Chapter 5 contains a theoretical development of fractal data encoding using a pyramidal approach. The results include an ultra-fast decoding method and a description of the relationship between the finite- and infinite-dimensional representation of the compressed data.

Chapter 6 describes the details of a fractal encoding scheme that matches or exceeds results obtainable using JPEG and some wavelet methods.

Chapter 7 and the next three chapters form a subsection of the book dedicated to results obtainable through a linear algebraic approach. This chapter sets up the model and gives simple, but previously elusive, conditions for convergence of the decoding process in the commonly used rms metric.

Chapter 8 derives a different ultrafast decoding scheme with the advantage of requiring a fixed number of decoding steps. This chapter also describes ways of overcoming some of the difficulties associated with encoding images as fractals.

Chapter 9 contains a theoretical treatment of a method to significantly reduce encoding times. The theoretical framework relates to other image compression methods (most notably VQ).

Chapter 10 contains a new approach to encoding images using the concepts of Chapters 7 and 8. This method overcomes the difficulty that standard fractal methods have in achieving very high fidelity.

Chapter 11 contains a theoretical treatment of fractal encoding with an emphasis on convergence.

Chapter 12 gives both a new model and an implementation of a fast encoding/decoding fractal method. This method is a direct IFS based solution to the image coding problem.

Chapter 13 contains a formulation of an image encoding method based on finite automata. The method generates highly compressed, resolution-independent encodings

The following appendices contain supplementary material.

Appendix A contains a listing of the code used to generate the results in Chapter 3, as well as an explanation of the code and a manual on its use.

Appendix B contains exercises that complement the main text. For the most part, these exercises are of the useful "show that such-and-such is true" rather than the uninformative "find something-or-other."

Appendix C contains a list of projects including video, parallelization, and new encoding and decoding methods.

Appendix D contains a brief comparison of the results in the book with JPEG and other methods.

Appendix E consists of the original images used in the text.

If the list of contributors has any conspicuous omissions, they are Michael Barnsley and Arnaud Jacquin. Barnsley (and his group, including D. Hardin, J. Elton, and A. Sloan) and Jacquin have probably done more innovative research in fractal image compression than anyone else in the field. Dr. Barnsley has his own book on the topic, and Dr. Jacquin declined to contribute to this book. Too bad.

Here is a brief editorial about fractal compression: Does fractal image compression have a role to play in the current rush to standardize video and still image compression methods? The fractal scheme suffers from two serious drawbacks: encoding is computationally intensive, and there is no "representation" theorem. The first means that even near-real time applications will require specialized hardware (for the foreseeable future); this is not the end of the world. The second is more serious; it means that unlike Fourier or wavelet methods, for example, the size of fractally encoded data gets very large as we attempt to approach perfect reconstruction. For example, a checkerboard image consisting of alternating black and white pixels cannot be encoded by any of the fractal schemes discussed in this book, except by the trivial (in the mathematical sense) solution of defining a map into each pixel of the image, leading to fractal image *expansion*.

Does this mean that fractal image compression is doomed? Probably not. In spite of the problems above, empirical results show that the fractal scheme is at least as good as, and better at some compression ranges, than the current standard, JPEG. Also, the scheme does possess several intriguing features. It is resolution independent; images can be reconstructed at any resolution, with the decoding process creating artificial data, when necessary, that is commensurate with the local behavior of the image data. This is currently something of a solution in search of a problem, but it may be useful. More importantly, the fractal scheme is computationally simple to decode. Software decoding of video, as well as still images, may be its saving grace.

The aim of this book is to show that a rich and interesting theory exists with results that are applicable. Even in the short amount of time devoted to this field, results are comparable with compression methods that have received hundreds of thousands, if not millions, more man-hours of research effort.

Finally, this book wouldn't have come into being without the support of my wife, Melinda. She said "sounds good to me," when anyone else would have said "what's that rattling sound," or "I smell something funny." She often says "sounds good to me" (as well as the other two things, now that I think of it), and I appreciate it.

I would also like to express my gratitude to the following people: my co-authors, whose contributions made this book possible; Barbara Burke, for editing my portion of the manuscript; and Elizabeth Sheehan, my calm editor at Springer-Verlag. My thanks also go to Henry Abarbanel, Hassan Aref, Andrew Gross, Arnold Mandel, Pierre Moussa, Rama Ramachandran, Dan Rogovin, Dan Salzbach, and Janice Shen, who, in one way or another, helped me along the way.

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Yuval Fisher, August 1994

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