

Frank Ihlenburg

Finite Element Analysis of Acoustic Scattering

With 88 Illustrations



Springer

Frank Ihlenburg
Germanischer Lloyd
Vorsetzen 32
D-20459 Hamburg
Germany

Editors

J.E. Marsden
Control and Dynamical Systems, 107-81
California Institute of Technology
Pasadena, CA 91125
USA

L. Sirovich
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA

Mathematics Subject Classification (1991): 65Nxx, 65Lxx, 76M10, 73Vxx

Library of Congress Cataloging-in-Publication Data

Ihlenburg, Frank.

Finite element analysis of acoustic scattering / Frank
Ihlenburg.

p. cm. — (Applied mathematical sciences ; 132)

Includes index.

ISBN 0-387-98319-8 (alk. paper)

1. Wave equations. 2. Scattering (Physics)—Mathematical models.
3. Helmholtz equation—Numerical solutions. 4. Boundary value
problems—Numerical solutions. 5. Finite element method.

I. Title. II. Series: Applied mathematical sciences (Springer-
Verlag New York Inc.) ; v. 132.

QA1.A647 vol. 132

[QC174.26.W28]

510 s—dc21

[531'.1133'01515353]

97-49493

© 1998 Springer-Verlag New York, Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

ISBN 0-387-98319-8 Springer-Verlag New York Berlin Heidelberg SPIN 10636277

Contents

Preface	vii
1 The Governing Equations of Time-Harmonic Wave Propagation	1
1.1 Acoustic Waves	1
1.1.1 Linearized Equations for Compressible Fluids	2
1.1.2 Wave Equation and Helmholtz Equation	3
1.1.3 The Sommerfeld Condition	6
1.2 Elastic Waves	8
1.2.1 Dynamic Equations of Elasticity	8
1.2.2 Vector Helmholtz Equations	9
1.3 Acoustic/Elastic Fluid–Solid Interaction	11
1.3.1 Physical Assumptions	12
1.3.2 Governing Equations and Special Cases	13
1.4 Electromagnetic Waves	16
1.4.1 Electric Fields	16
1.4.2 Magnetic Fields	17
1.4.3 Maxwell’s Equations	18
1.5 Summary	19
1.6 Bibliographical Remarks	20
2 Analytical and Variational Solutions of Helmholtz Problems	21
2.1 Separation of Variables	22

2.1.1	Cartesian Coordinates	22
2.1.2	Spherical Coordinates	24
2.1.3	Cylindrical Coordinates	29
2.1.4	Atkinson–Wilcox Expansion	31
2.1.5	Far-Field Pattern	32
2.1.6	Computational Aspects	32
2.2	References from Functional Analysis	35
2.2.1	Norm and Scalar Product	35
2.2.2	Hilbert Spaces	36
2.2.3	Sesquilinear Forms and Linear Operators	38
2.2.4	Trace of a Function	39
2.3	Variational Formulation of Helmholtz Problems	40
2.3.1	Helmholtz Problems on Bounded Domains	40
2.3.2	Helmholtz Problems on Unbounded Domains	41
2.3.3	Weak Formulation for Solid–Fluid Interaction	43
2.4	Well-Posedness of Variational Problems	46
2.4.1	Positive Definite Forms	46
2.4.2	The inf–sup Condition	48
2.4.3	Coercive Forms	51
2.4.4	Regularity and Stability	53
2.5	Variational Methods	53
2.5.1	Galerkin Method and Ritz Method	53
2.5.2	Convergence Results	55
2.5.3	Conclusions for Helmholtz Problems	57
2.6	Summary	58
2.7	Bibliographical Remarks	58
3	Discretization Methods for Exterior Helmholtz Problems	61
3.1	Decomposition of Exterior Domains	62
3.1.1	Introduction of an Artificial Boundary	62
3.1.2	Dirichlet-to-Neumann Operators	63
3.1.3	Well-Posedness	64
3.2	The Dirichlet-to-Neumann Operator and Numerical Applications	65
3.2.1	The Exact DtN Operator	65
3.2.2	Spectral Characterization of the DtN-Operator	67
3.2.3	Truncation of the DtN Operator	69
3.2.4	Localizations of the Truncated DtN Operator	70
3.3	Absorbing Boundary Conditions	71
3.3.1	Recursion in the Atkinson–Wilcox Expansion	72
3.3.2	Localization of a Pseudodifferential Operator	74
3.3.3	Comparison of ABC	76
3.3.4	The PML Method	78
3.4	The Finite Element Method in the Near Field	80
3.4.1	Finite Element Technology	81

3.4.2	Identification of the FEM as a Galerkin Method . . .	86
3.4.3	The h -Version and the hp -Version of the FEM . . .	87
3.5	Infinite Elements and Coupled Finite-Infinite Element Discretization	87
3.5.1	Infinite Elements from Radial Expansion	87
3.5.2	Variational Formulations	89
3.5.3	Remarks on the Analysis of the Finite-Infinite Element Method	93
3.6	Summary	97
3.7	Bibliographical Remarks	98

4 Finite Element Error Analysis and Control for Helmholtz Problems 101

4.1	Convergence of Galerkin FEM	102
4.1.1	Error Function and Residual	103
4.1.2	Positive Definite Problems	103
4.1.3	Indefinite Problems	105
4.2	Model Problems for the Helmholtz Equation	106
4.2.1	Model Problem I: Uniaxial Propagation of a Plane Wave	107
4.2.2	Model Problem II: Propagation of Plane Waves with Variable Direction	108
4.2.3	Model Problem III: Uniaxial Fluid-Solid Interaction	109
4.3	Stability Estimates for Helmholtz Problems	110
4.3.1	The inf-sup Condition	110
4.3.2	Stability Estimates for Data of Higher Regularity . .	113
4.4	Quasioptimal Convergence of FE Solutions to the Helmholtz Equation	116
4.4.1	Approximation Rule and Interpolation Error	116
4.4.2	An Asymptotic Error Estimate	119
4.4.3	Conclusions	121
4.5	Preasymptotic Error Estimates for the h -Version of the FEM	122
4.5.1	Dispersion Analysis of the FE Solution	122
4.5.2	The Discrete inf-sup Condition	124
4.5.3	A Sharp Preasymptotic Error Estimate	125
4.5.4	Results of Computational Experiments	128
4.6	Pollution of FE Solutions with Large Wave Number	132
4.6.1	Numerical Pollution	133
4.6.2	The Typical Convergence Pattern of FE Solutions to the Helmholtz Equation	134
4.6.3	Influence of the Boundary Conditions	136
4.6.4	Error estimation in the L^2 -norm	137
4.6.5	Results from 2-D Computations	138
4.7	Analysis of the hp FEM	140
4.7.1	hp -Approximation	140

4.7.2	Dual Stability	145
4.7.3	FEM Solution Procedure. Static Condensation	147
4.7.4	Dispersion Analysis and Phase Lag	149
4.7.5	Discrete Stability	151
4.7.6	Error Estimates	153
4.7.7	Numerical Results	155
4.8	Generalized FEM for Helmholtz Problems	158
4.8.1	Generalized FEM in One Dimension	158
4.8.2	Generalized FEM in Two Dimensions	162
4.9	The Influence of Damped Resonance in Fluid–Solid Interaction	170
4.9.1	Analysis and Parameter Discussion	170
4.9.2	Numerical Evaluation	171
4.10	A Posteriori Error Analysis	174
4.10.1	Notation	174
4.10.2	Bounds for the Effectivity Index	175
4.10.3	Numerical Results	179
4.11	Summary and Conclusions for Computational Application .	185
4.12	Bibliographical Remarks	187
5	Computational Simulation of Elastic Scattering	189
5.1	Elastic Scattering from a Sphere	189
5.1.1	Implementation of a Coupled Finite–Infinite Element Method for Axisymmetric Problems	189
5.1.2	Model Problem	191
5.1.3	Computational Results	194
5.1.4	Conclusions	201
5.2	Elastic Scattering from a Cylinder with Spherical Endcaps .	202
5.2.1	Model Parameters	202
5.2.2	Convergence Tests	203
5.2.3	Comparison with Experiments	206
5.3	Summary	210
	References	211
	Index	221