Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1309

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Global Bifurcation of Periodic Solutions with Symmetry

Springer-Verlag
Berlin Heidelberg New York London Paris Tokyo

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Mathematics Subject Classification (1980): 34XX, 35XX, 47H15, 58C27, 58F14, 58F22, 76E30, 80A30, 80A32

ISBN 3-540-19234-4 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-19234-4 Springer-Verlag New York Berlin Heidelberg

Library of Congress Cataloging-in-Publication Data. Fiedler, Bernold, 1956- Global bifurcation of periodic solutions with symmetry / Bernold Fiedler. p. cm.–(Lecture notes in mathematics; 1309) Bibliography: p. Includes index.
ISBN 0-387-19234-4 (U.S.)
1. Bifurcation theory. 2. Singularities (Mathematics) 3. Nonlinear operators. 4. Differential equations–Numerical solutions. 5. Differential equations, Partial–Numerical solutions. I. Title. II. Title: Periodic solutions with symmetry. III. Series: Lecture notes in mathematics (Springer-Verlag); 1309.
QA3.L28 no. 1309 [QA372] 510 s–dc 19 [514'.74] 88-12342

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Printed in Germany

Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.
2146/3140-543210

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Preface

The inherent harmony of periodic motions as well as of symmetry has exerted its own fascination, as it seems, ever since the dawn of thought. Today, such a "harmonia mundi" is at least hoped for on just about any possible scale: from elementary particle physics to astronomy.

In search of some harmony let us ask naive questions. Suppose we are given a dynamical system with some built-in symmetry. Should we expect periodic motions which somehow reflect this symmetry? And how would periodicity harmonize with symmetry?

These almost innocent questions are the entrance to a labyrinth of intricacies. Probing only along some fairly safe threads we are lead from dynamics to topology, algebra, singularity theory, numerical analysis, and to some applications. A global point of view will be one guiding theme along our way: we are mainly interested in periodic motions far from equilibrium.

For a method we rely on bifurcation theory, on transversality theory, and on generic approximations. As a reward we encounter known local singularities. As a central new aspect we study the global interaction and interdependence of these local singularities, designing a homotopy invariant. As a result, we obtain an index $\mathcal{H}$ which evaluates only information at stationary solutions. Nonzero $\mathcal{H}$ implies global Hopf bifurcation of periodic solutions with certain symmetries. Putting it emphatically, $\mathcal{H}$ harmonizes symmetry and periodicity . Curiously, $\mathcal{H}$ need not be homotopy invariant. It is one of my favorite speculations that this obstruction may hint at chaotic motions.

Cyclic motions relate to cyclic groups. Phrasing this relation between dynamics and algebra less sloppily: the symmetry of a periodic solution of a dynamical system is related to a cyclic factor within the group of symmetries of that system. Curiously, some period doubling bifurcations relate to the number 2, acting by multiplication on such a cyclic group. The multiplicative order of 2 relates to the number of possibly different indices $\mathcal{H}$ for a given system.

Symmetry, although beautiful, causes numerical difficulties. Basically, groups with irreducible representations of higher dimensions entail higher local singularities which are not very well understood. This is an obstacle to numerical pathfollowing algorithms. We will give a complete list of the easier, lower-dimensional generic bifurcations. Avoiding cyclic loops in the associated global bifurcation diagrams by a suitable homotopy invariant will be a central issue in our theoretical analysis. Both aspects are essential prerequisites for an efficient numerical pathfollowing method in dynamical systems with symmetries.

In real applications, as in real life, the lofty regions of harmony, periodicity, and symmetry are always confronted with the abysmal danger of destabilization. Surprisingly, there are still some applications where periodicity and symmetry is observed. We will concentrate on chemical waves as a model example below, though the theory is general. We obtain rotating waves (spirals) in continuous geometries, and phase-locked oscillations in discrete geometries.

Because it may not at all be easily detected by the reader, let me confess here a guiding principle for this book. Like so many others, I have tried to dismiss difficulty for beauty.

I happily say my thanks to everyone who has helped me. In particular, I would like to mention J. Alexander, G. Auchmuty, T. Bartsch, A. Brandis, S.-N. Chow, R. Cushman, R. Field, S. v. Gils, M. Golubitsky, W. Jäger, P. Kunkel, R. Lauterbach, J. Mallet-Paret, M. Marek, M. Medveď, J.-C. van der Meer, C. Pospiech, J. Sanders, D. Sattinger, R. Schaaf, A. Vanderbauwhede, A. Wagner, J. Yorke, and all those friends who have helped with proofreading. Typesetting the whole manuscript in TeX was a laborious task. It was performed by M. Torterolo with great patience. Finally, I am indebted to Springer-Verlag for an efficient and pleasant cooperation.

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