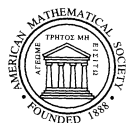


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# Fundamental Algebraic Geometry

Grothendieck's FGA Explained

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