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Dirichlet Forms

Lectures given at the 1st Session of the Centro
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Preface

In recent years the theory of Dirichlet forms has witnessed some very important developments both in its theoretical foundations and in its applications, which have ranged from the theory of Stochastic Processes to Quantum Field Theory to the theory of highly inhomogeneous materials.

It was therefore felt that it was due time to have on this subject a CIME school, in which leading experts in the field would present both the basic theoretical aspects and some of the recent applications, pointing to areas of active research.

The school was organized in six courses, covering

- Foundations of the theory and connection with Potential Theory and Stochastic Processes (M. Fukushima, M. Roeckner)
- Regularity results and a-priori estimates for solutions of elliptic equations in general domains (E. Fabes, C. Kenig)
- Hypercontractivity of semigroups and relation with spectral properties (L. Gross)
- Logarithmic Sobolev inequalities and connections with Statistical Mechanics (D. Stroock)

In the afternoons of the last three days a workshop was held, with financial support from the CNR project "Irregular Variational Problems", on further applications of the theory of Dirichlet Forms, also in the noncommutative setting.

The seminars were given by

- . L. Accardi (Roma II) Noncommutative Stochastic Processes and applications
- . S. Albeverio (Bochum) Construction of infinite-dimensional processes
- . M. Biroli (Pol. Milano) Asymptotic Dirichlet Forms
- . F. Guerra (Roma I) Annealing and Cavitation in Spin Glass Models
- . M. Lindsay (Nottingham) Noncommutative Semigroups
- . S.R.S. Varadhan (Courant Institute) Hydrodynamic Limit and Entropy Estimates I, II
- . Ma Zhiming (Academia Sinica) Nonsymmetric Dirichlet Forms

The lectures of the School were followed with constant and active participation by many young researchers, both from Italy and from abroad.

We believe that the School was successful in reaching its aims, and we wish to express here our appreciation to the speakers for the high quality of their lectures and for their availability for discussions during the School.

We would also like to thank prof. R. Conti and the CIME Scientific Committee for the invitation to organize the School, and prof. P. Zecca and the staff of the Centro Volta in Como for their very effective help.

Gianfausto Dell'Antonio
Umberto Mosco

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