Heinz-Otto Peitgen Hartmut Jürgens Dietmar Saupe

Fractals for the Classroom

Part Two Complex Systems and Mandelbrot Set

Evan Maletsky Terry Perciante Lee Yunker NCTM Advisory Board



National Council of Teachers of Mathematics



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